Trading between task-level and data-level parallelism to solve sparse triangular linear systems

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Circuits and Systems Group
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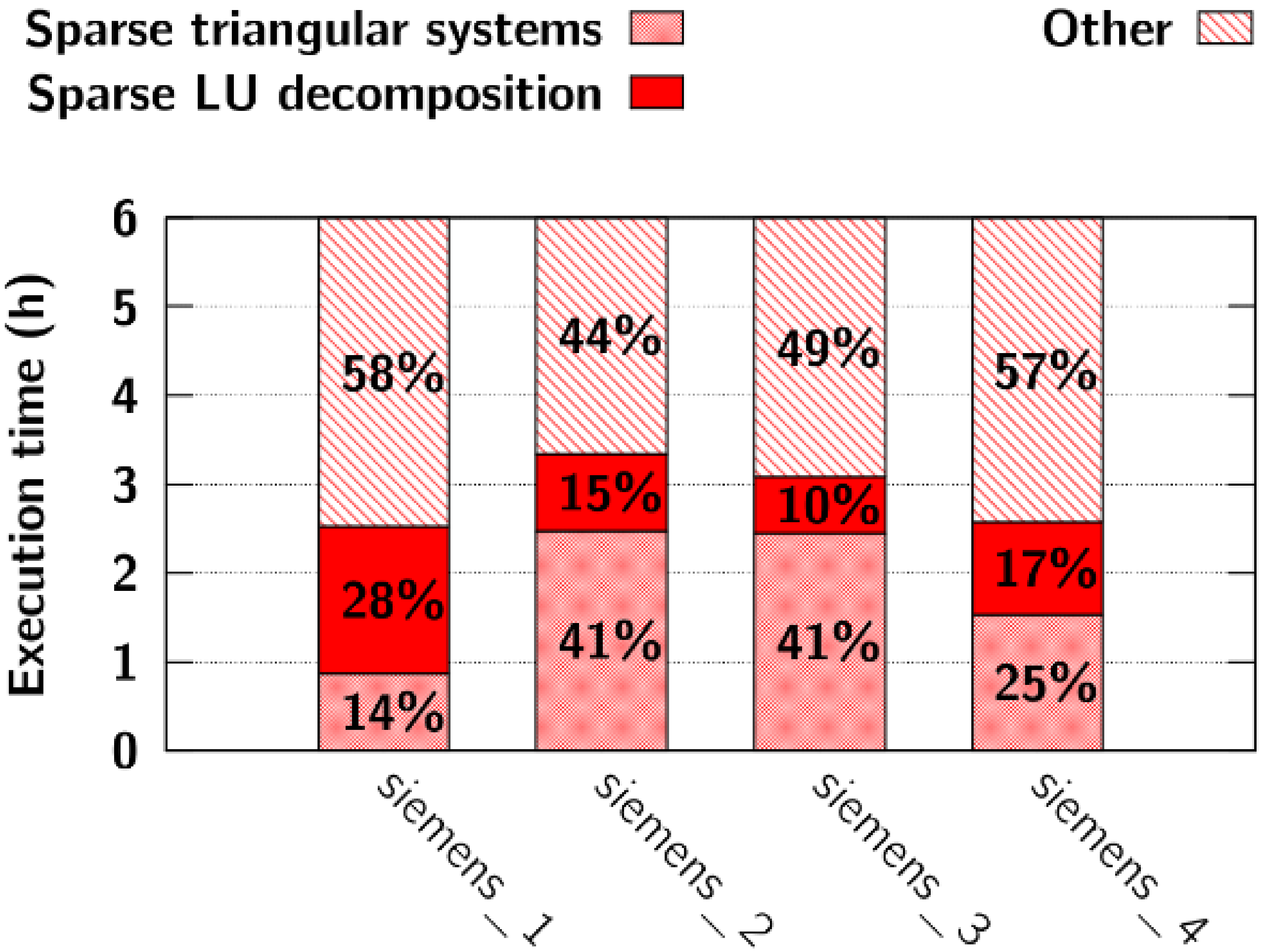
12 October 2015



Data dependencies in sparse triangular systems are represented using directed acyclic graphs

$$\begin{cases} x_1 = b_1 \\ l_{2,1}x_1 + x_2 = b_2 \\ l_{3,1}x_1 + x_3 = b_3 \\ x_4 = b_4 \\ l_{5,4}x_4 + x_5 = b_5 \\ l_{6,3}x_3 + l_{6,5}x_5 + x_6 = b_6 \\ l_{7,6}x_6 + x_7 = b_7 \\ l_{8,6}x_6 + x_8 = b_8 \end{cases}$$

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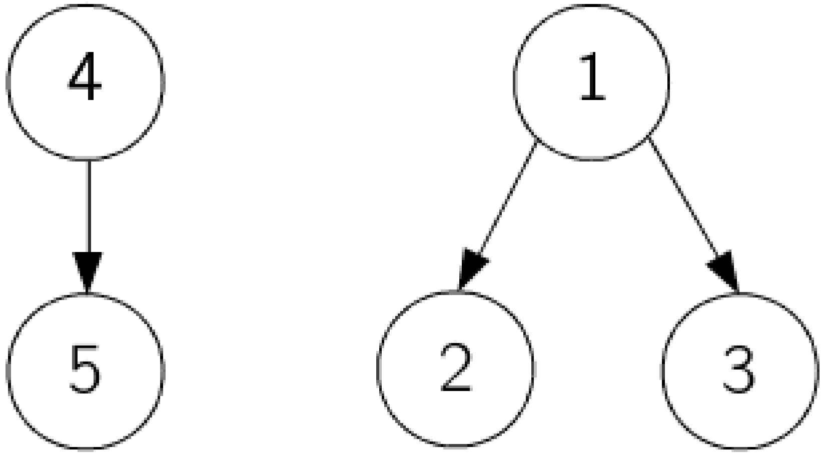

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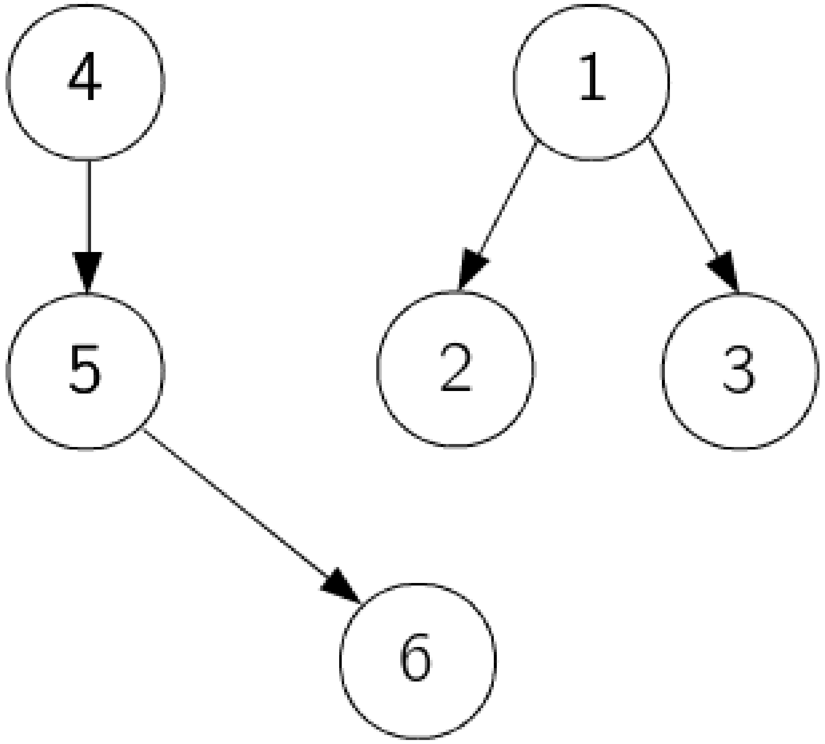

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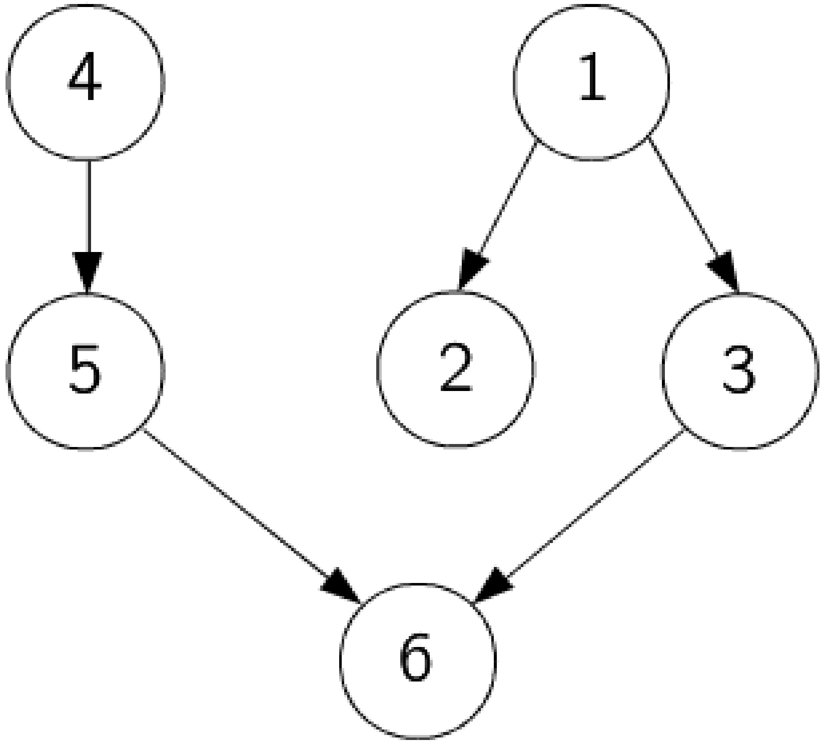

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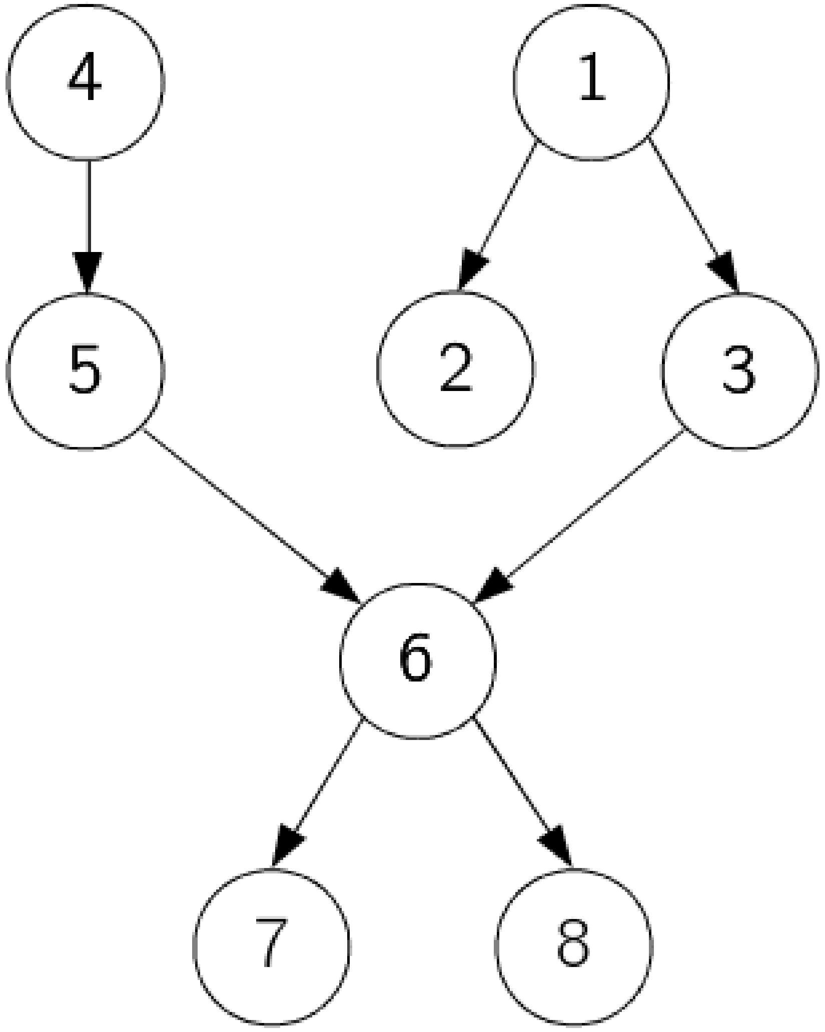

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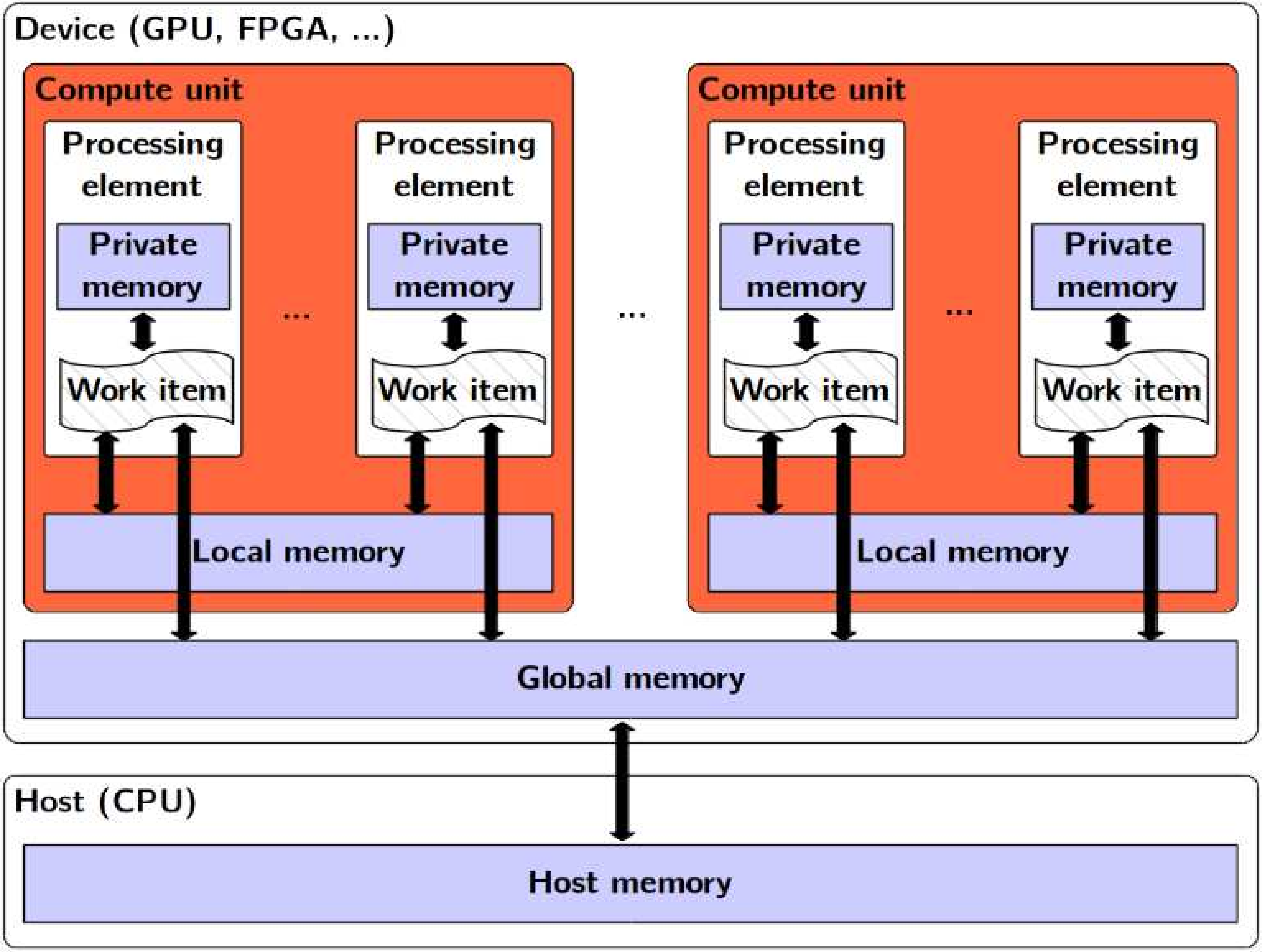
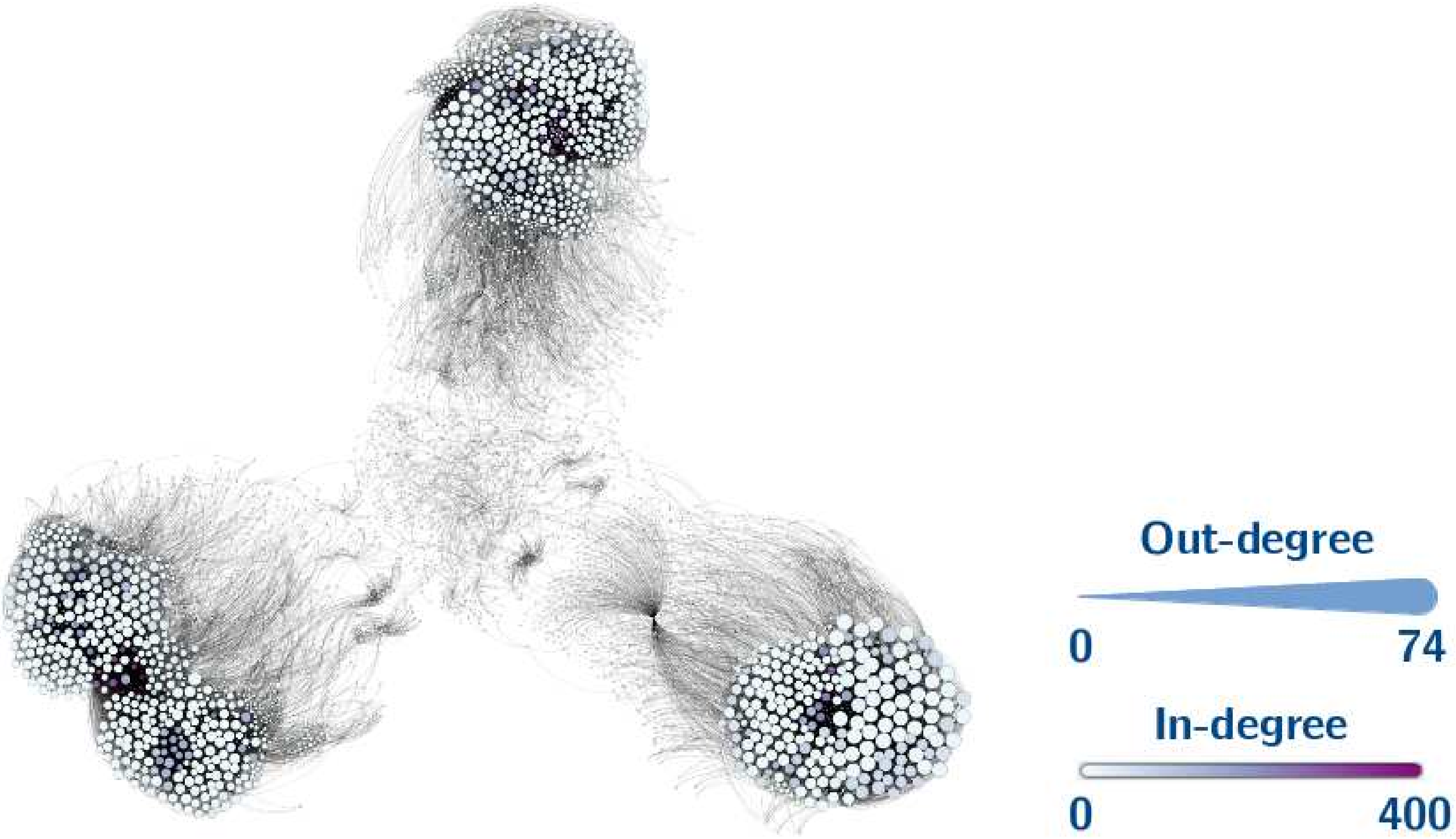
GPUs are composed of compute units, each one containing a local memory



Our approach is to partition the graph into sub-graphs and process each one inside a compute unit



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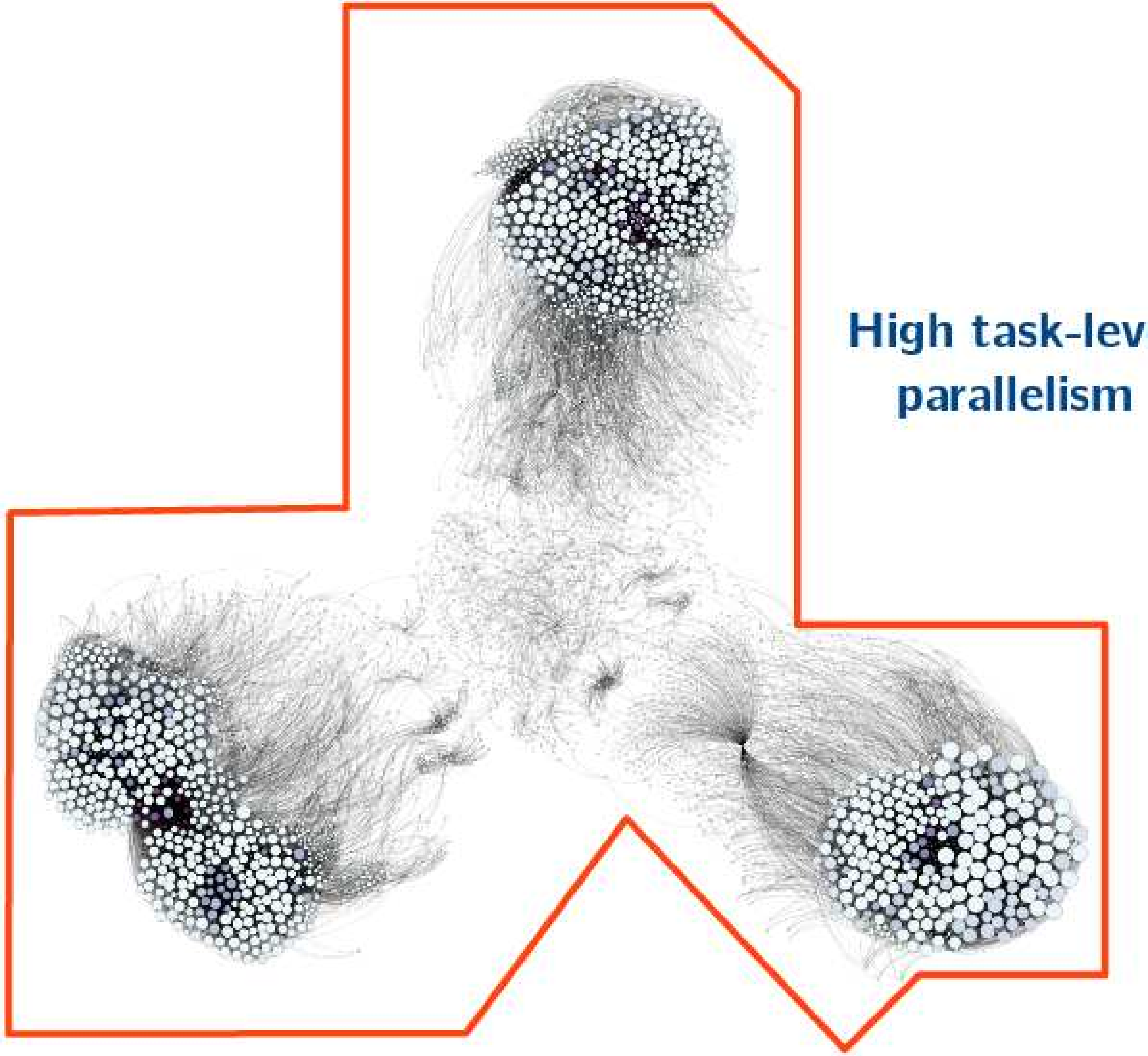
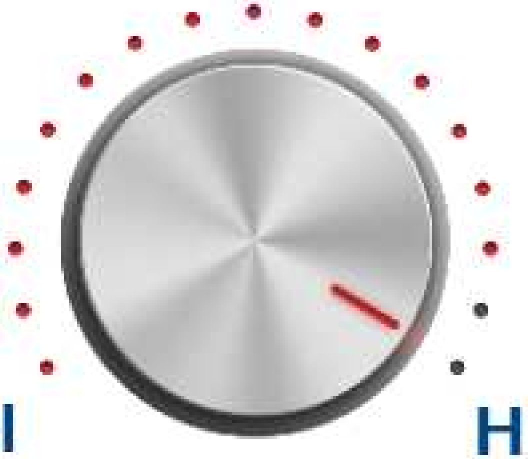
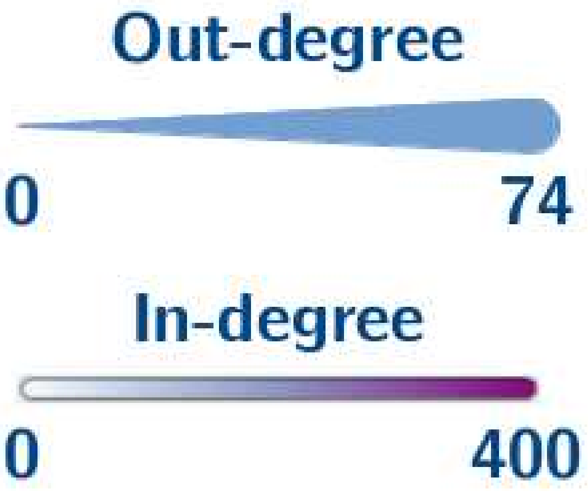




High task-level parallelism

High data-level parallelism



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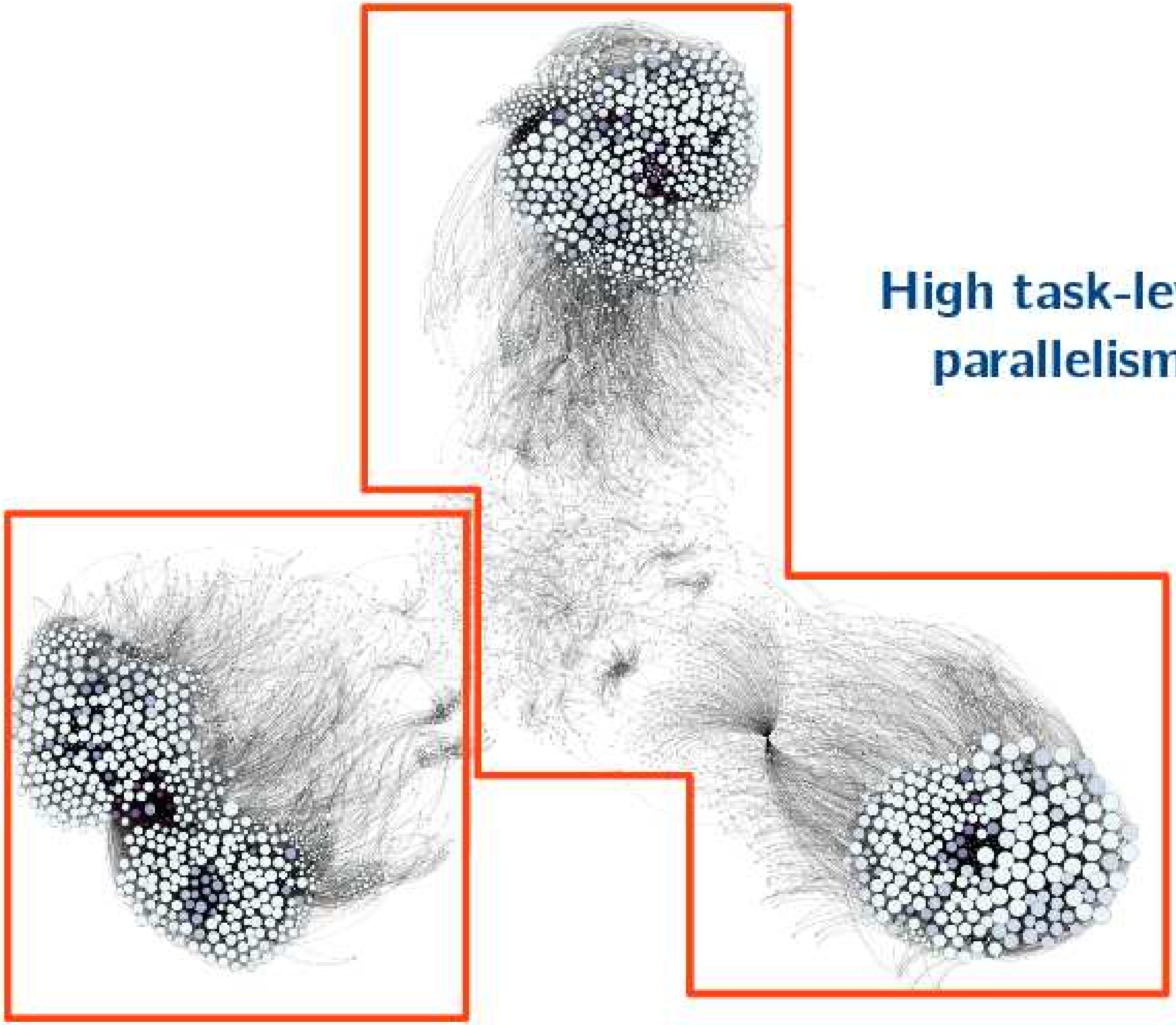

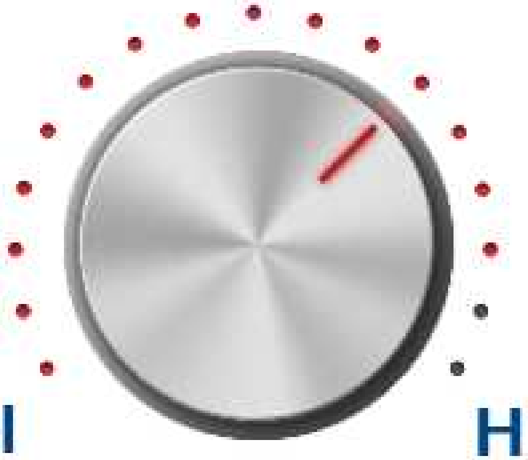

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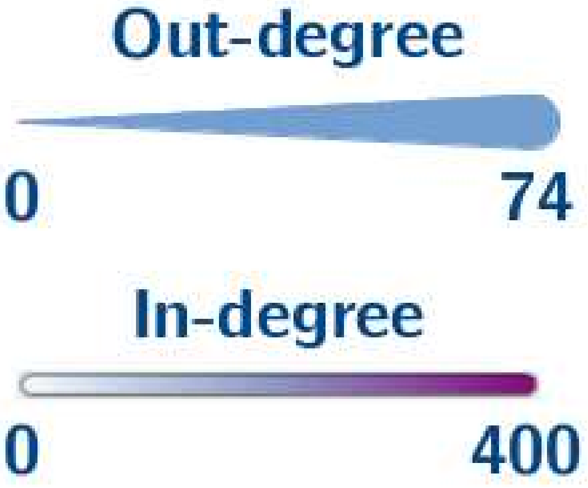

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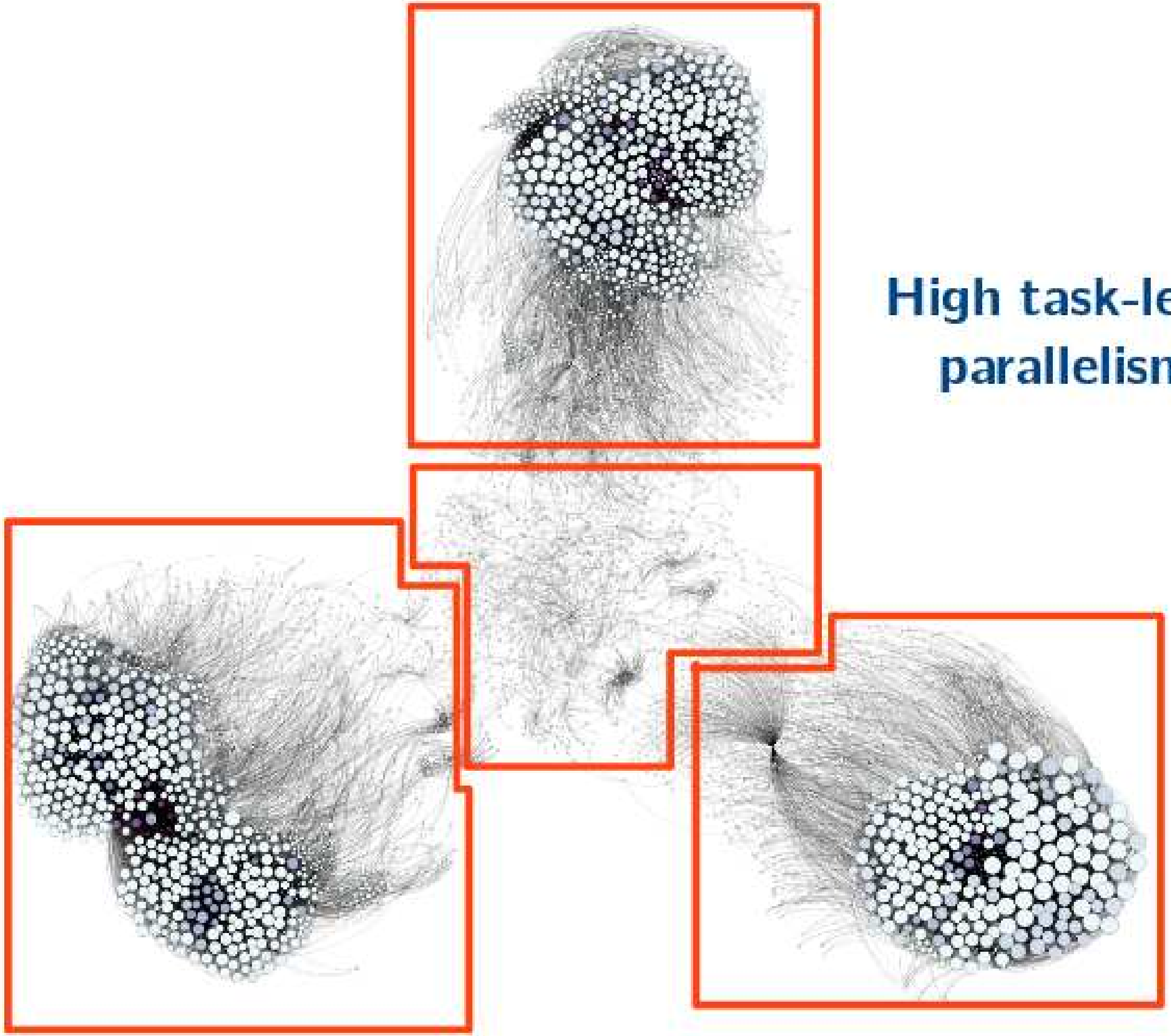

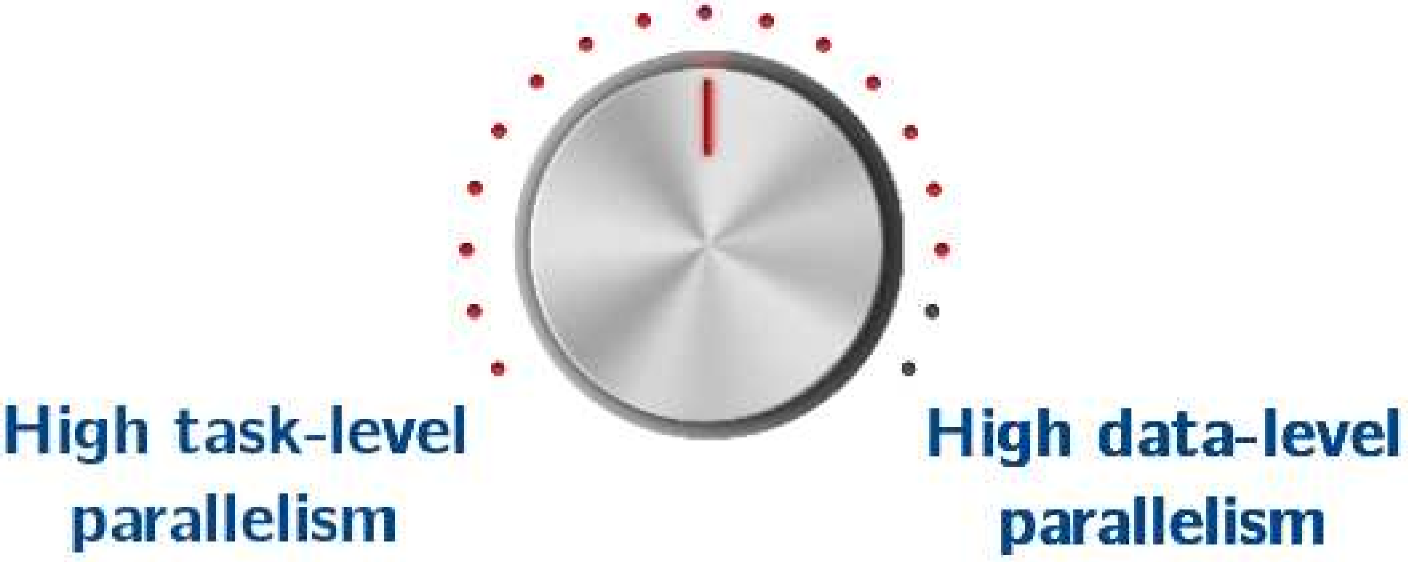

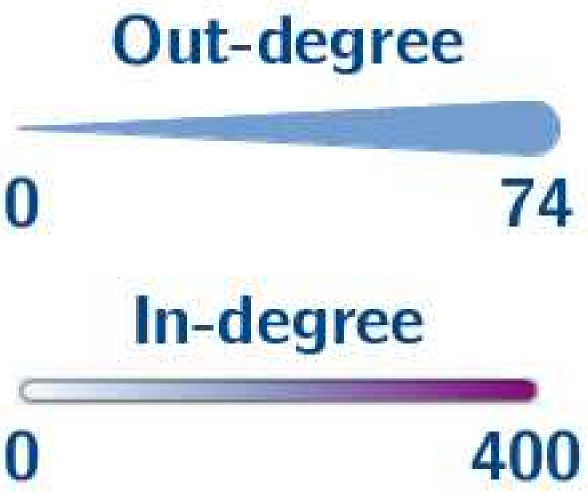

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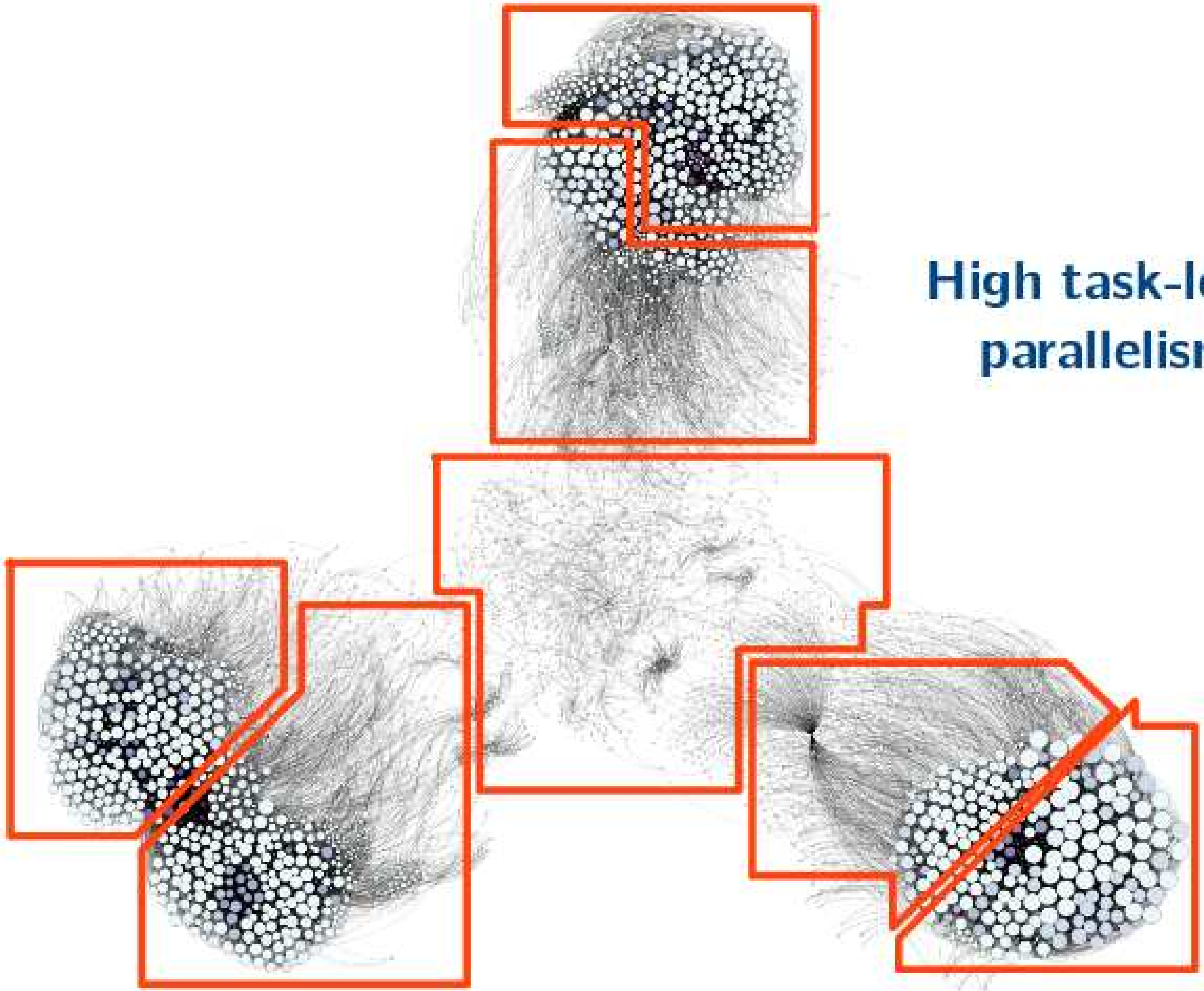

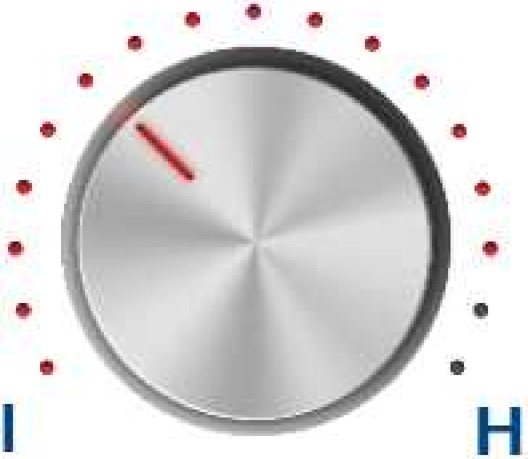

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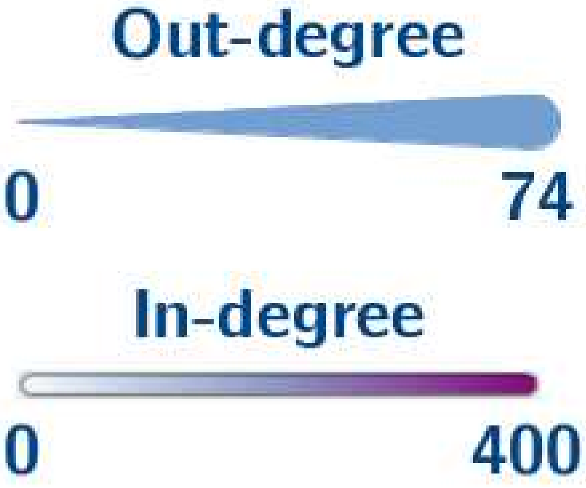

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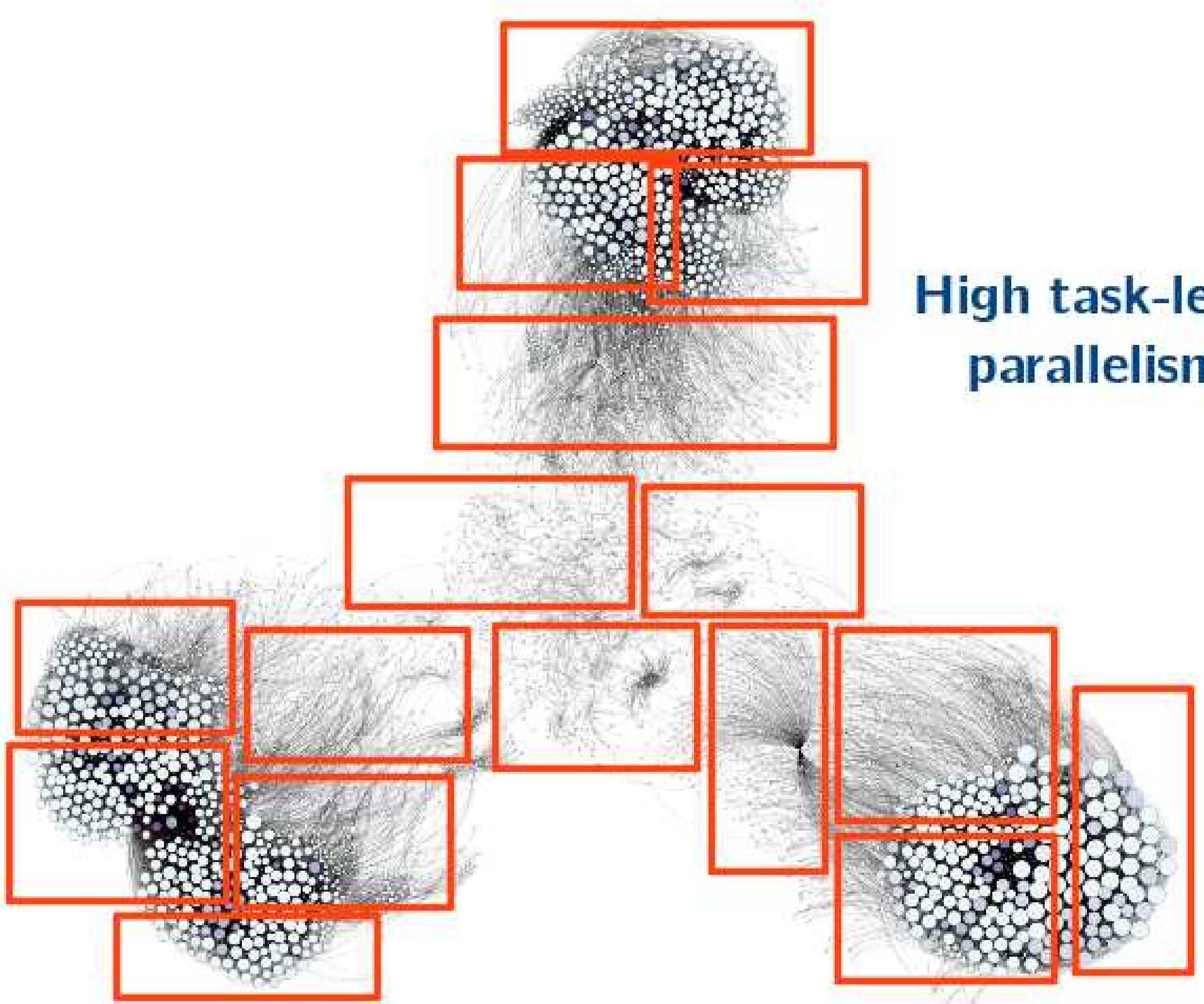
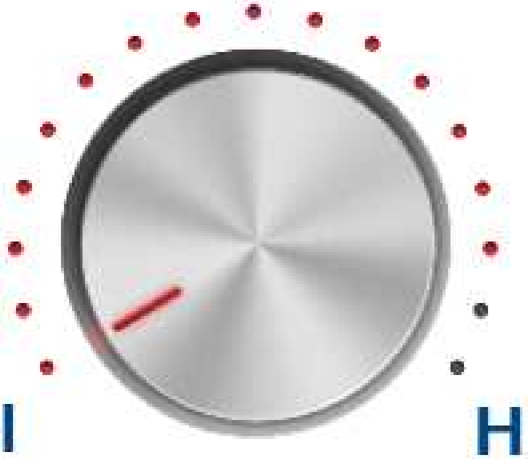
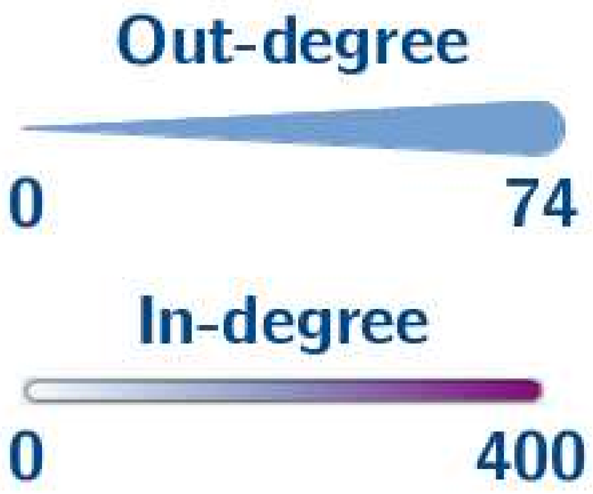
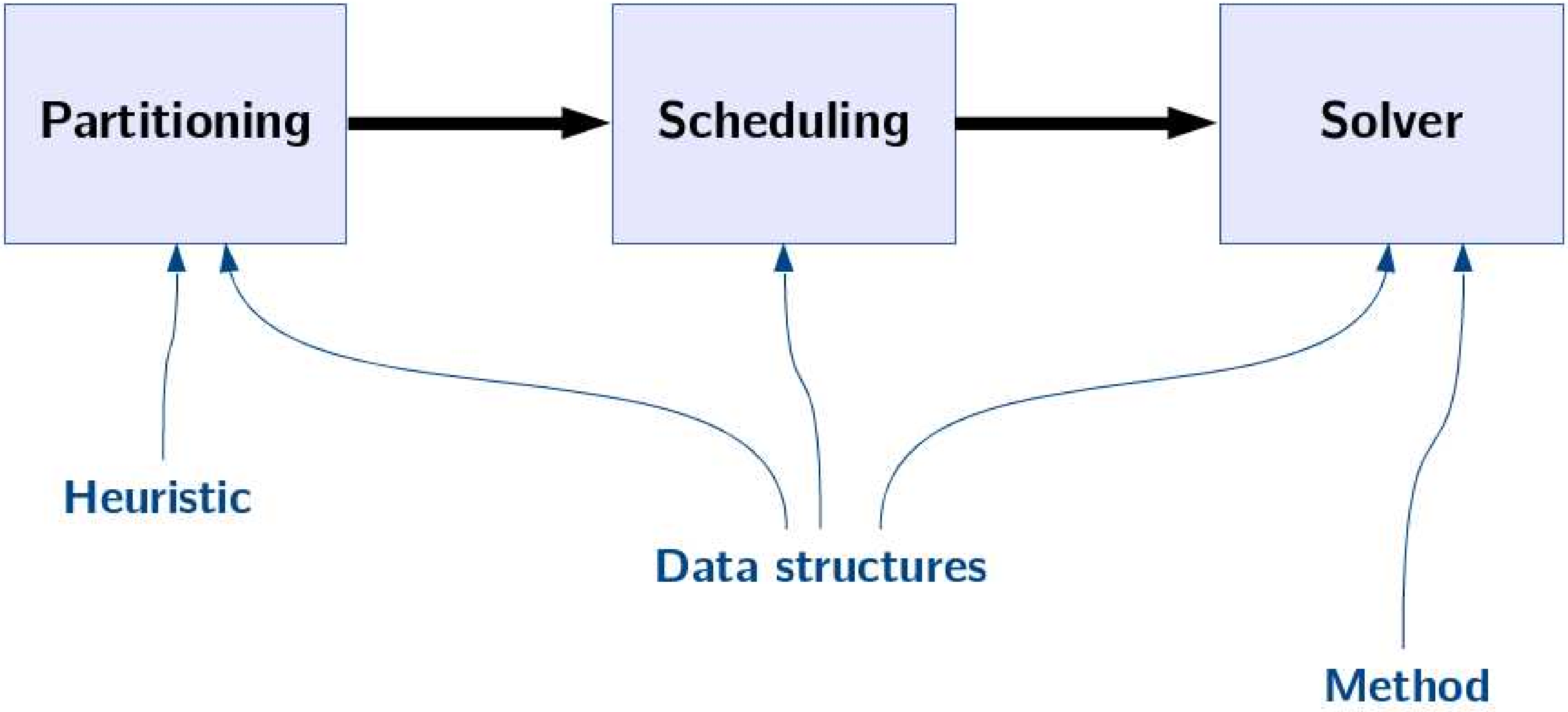






Our technique is divided in three phases



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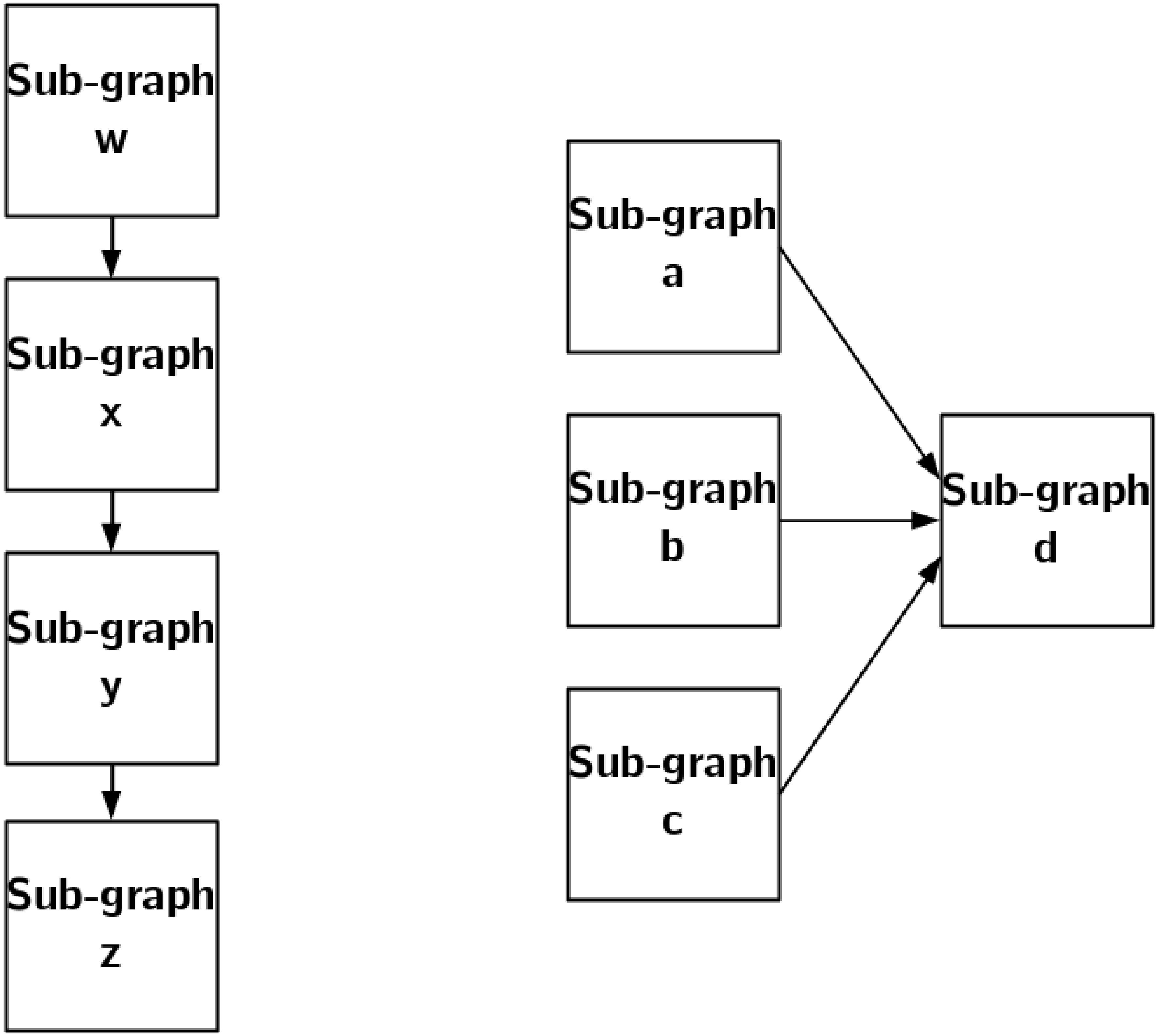
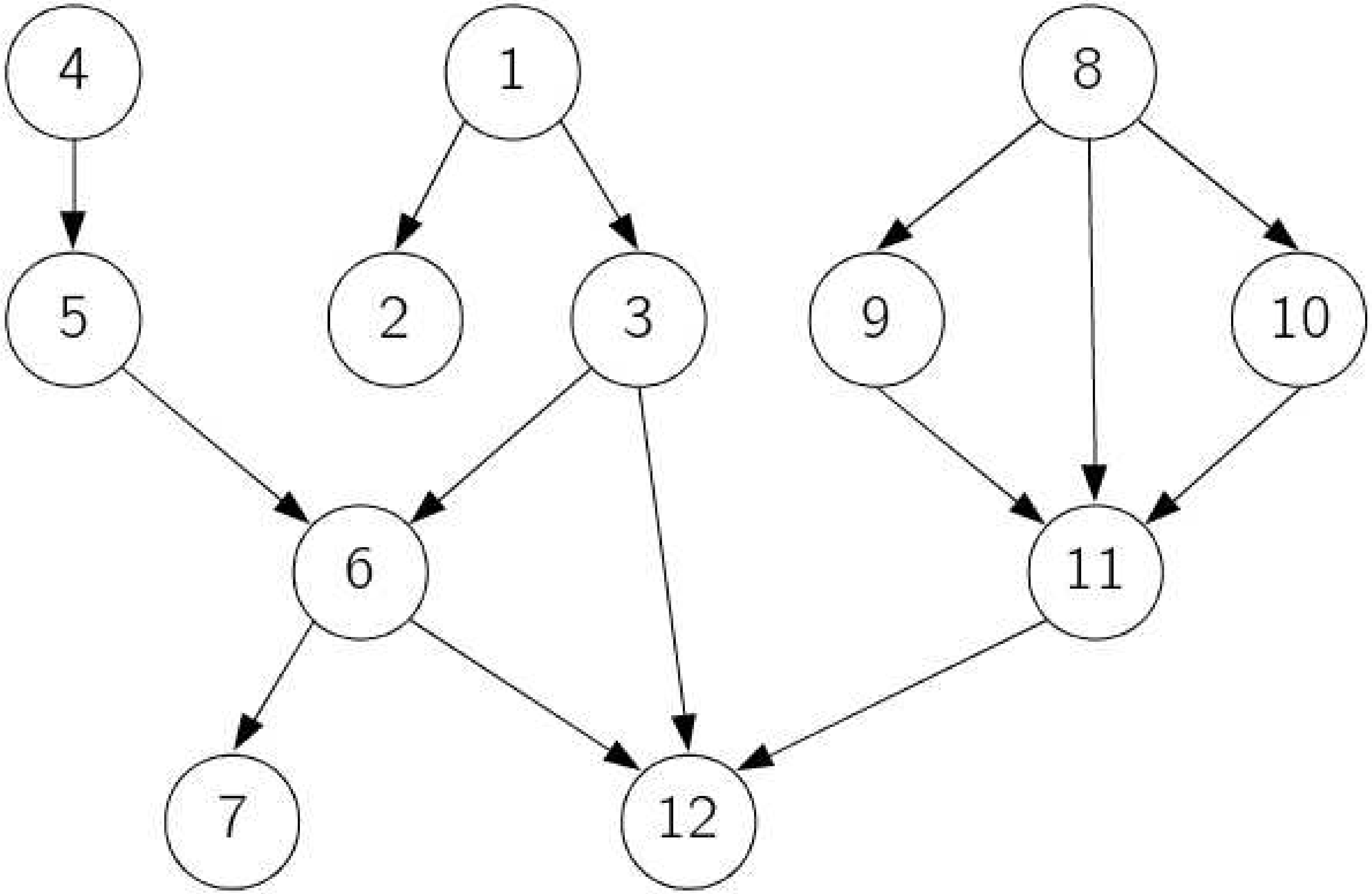
Our partitioning heuristic aims to achieve a specific dependency pattern for sub-graphs



At each step of the algorithm, we consider the vertices in order of their out-degree

Local memory size = 3

Priority rule: 🟥 1 🟨 2 🟦 3 🟩 4



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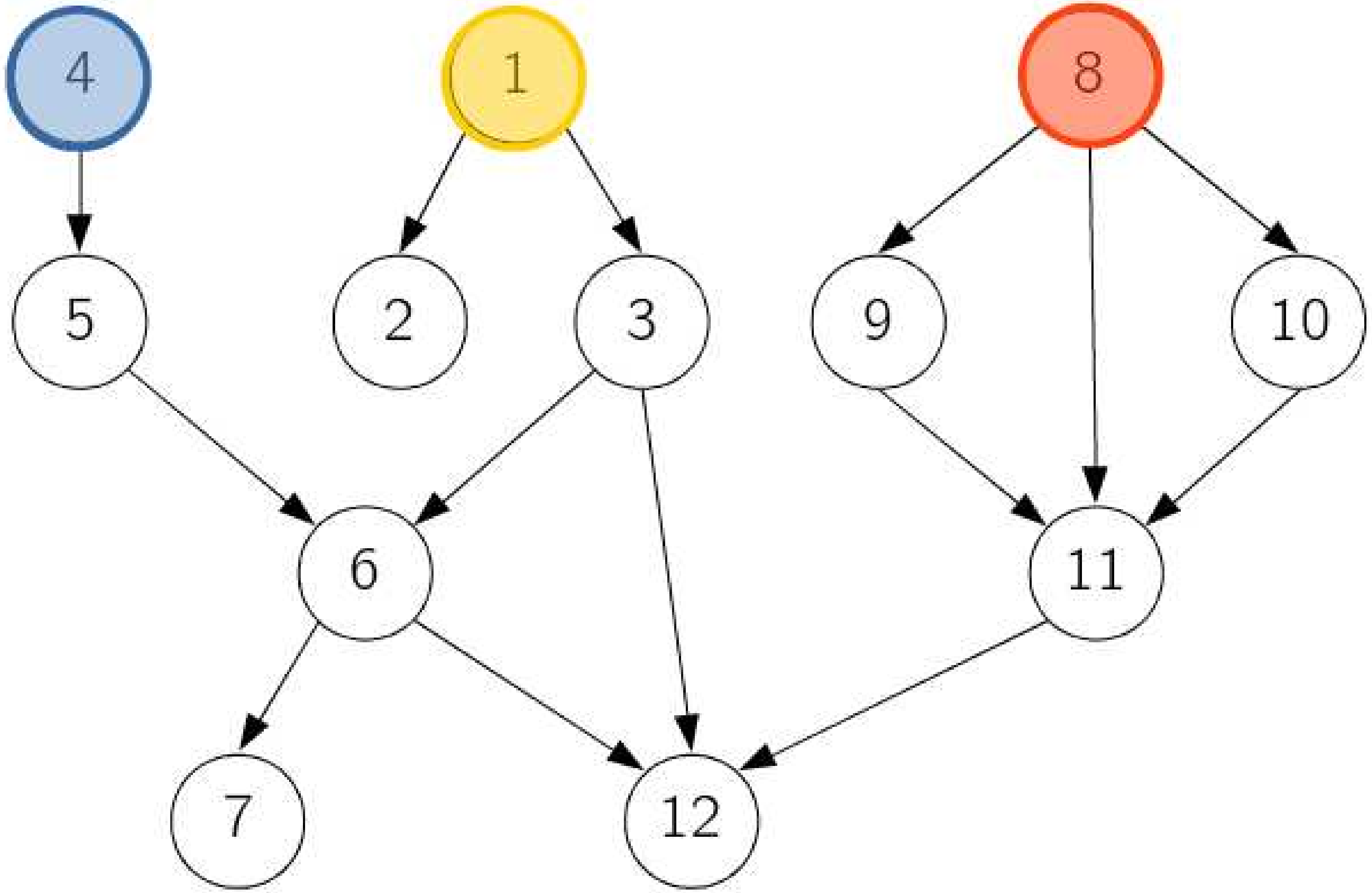

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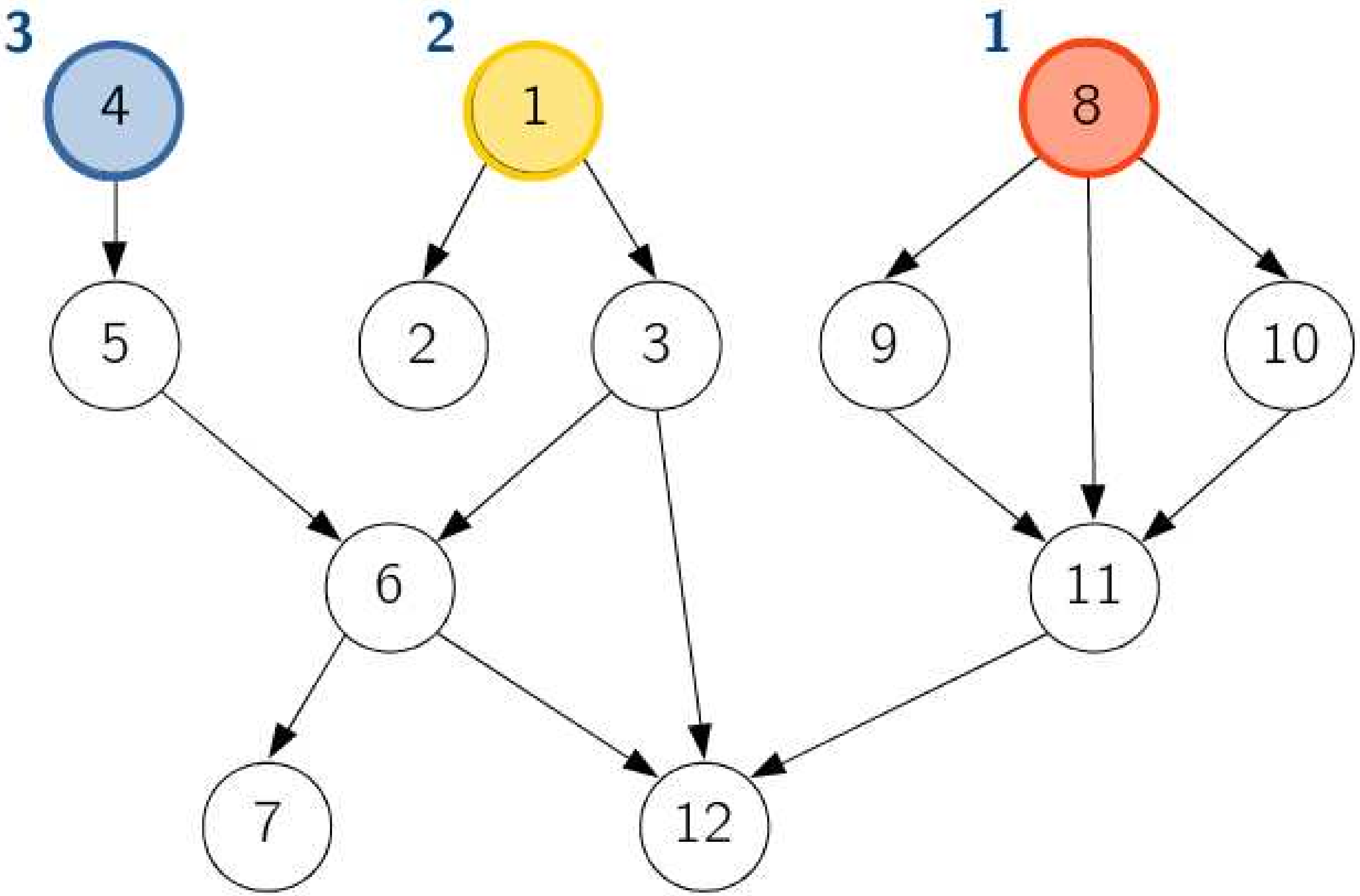

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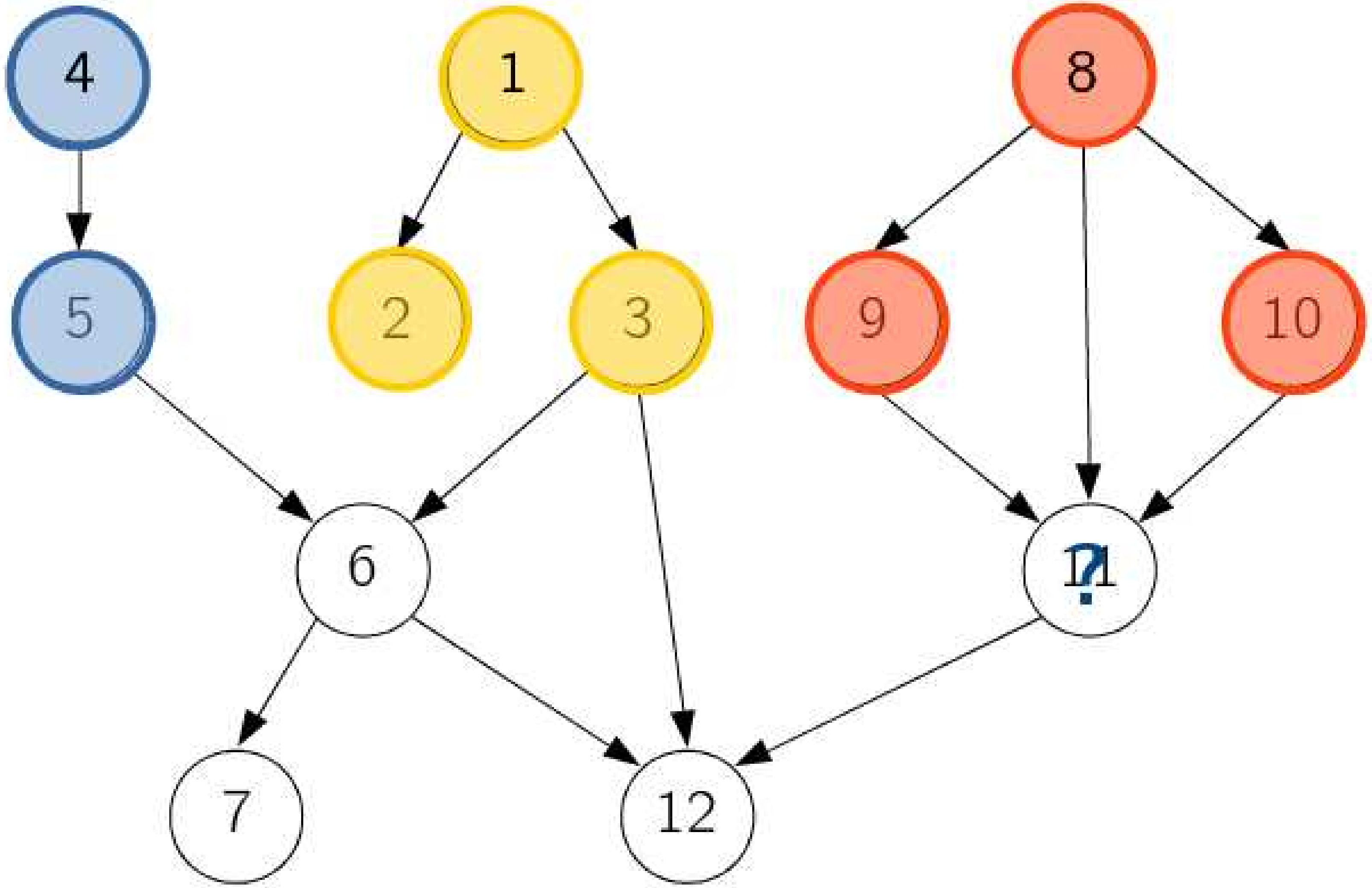

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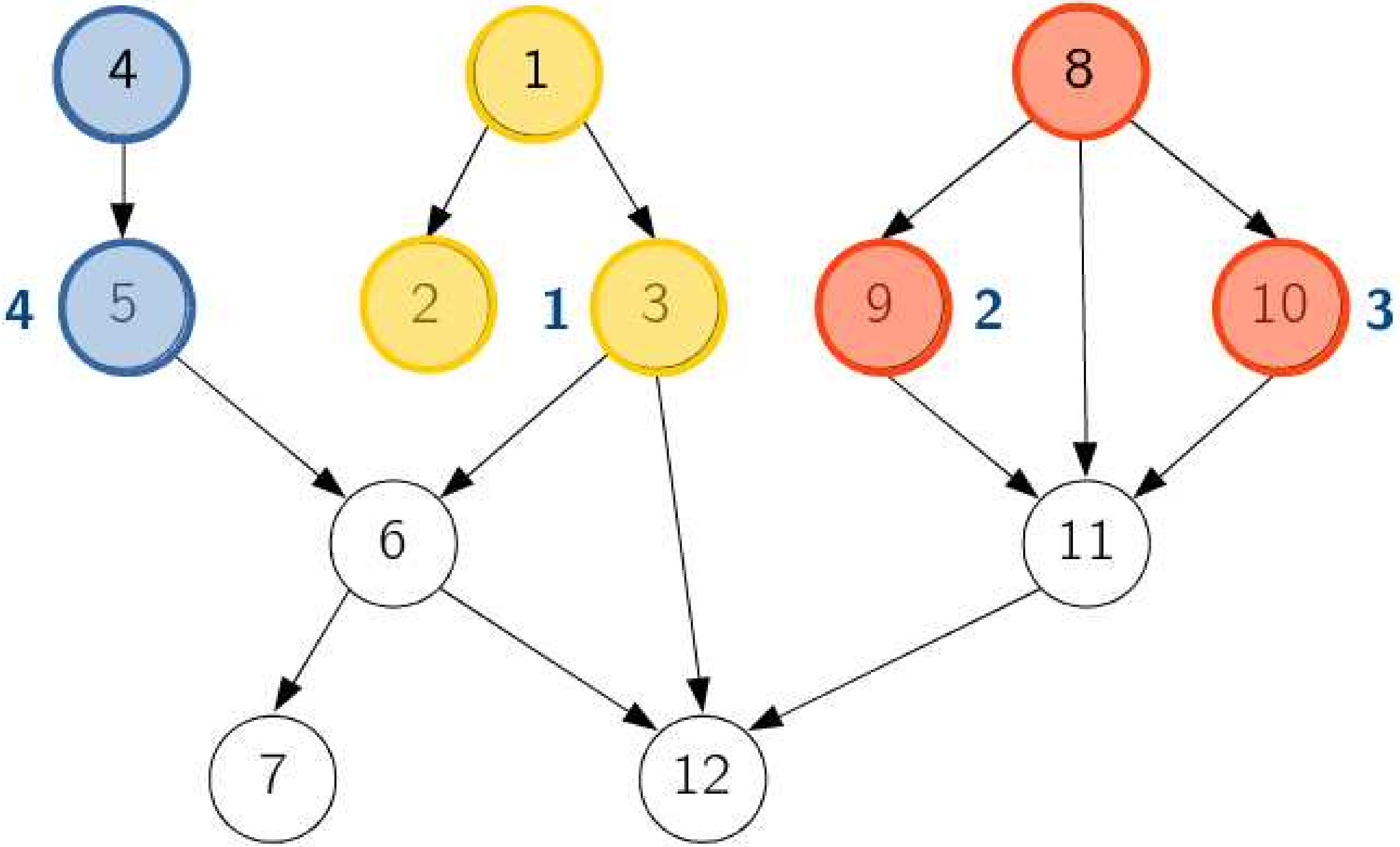

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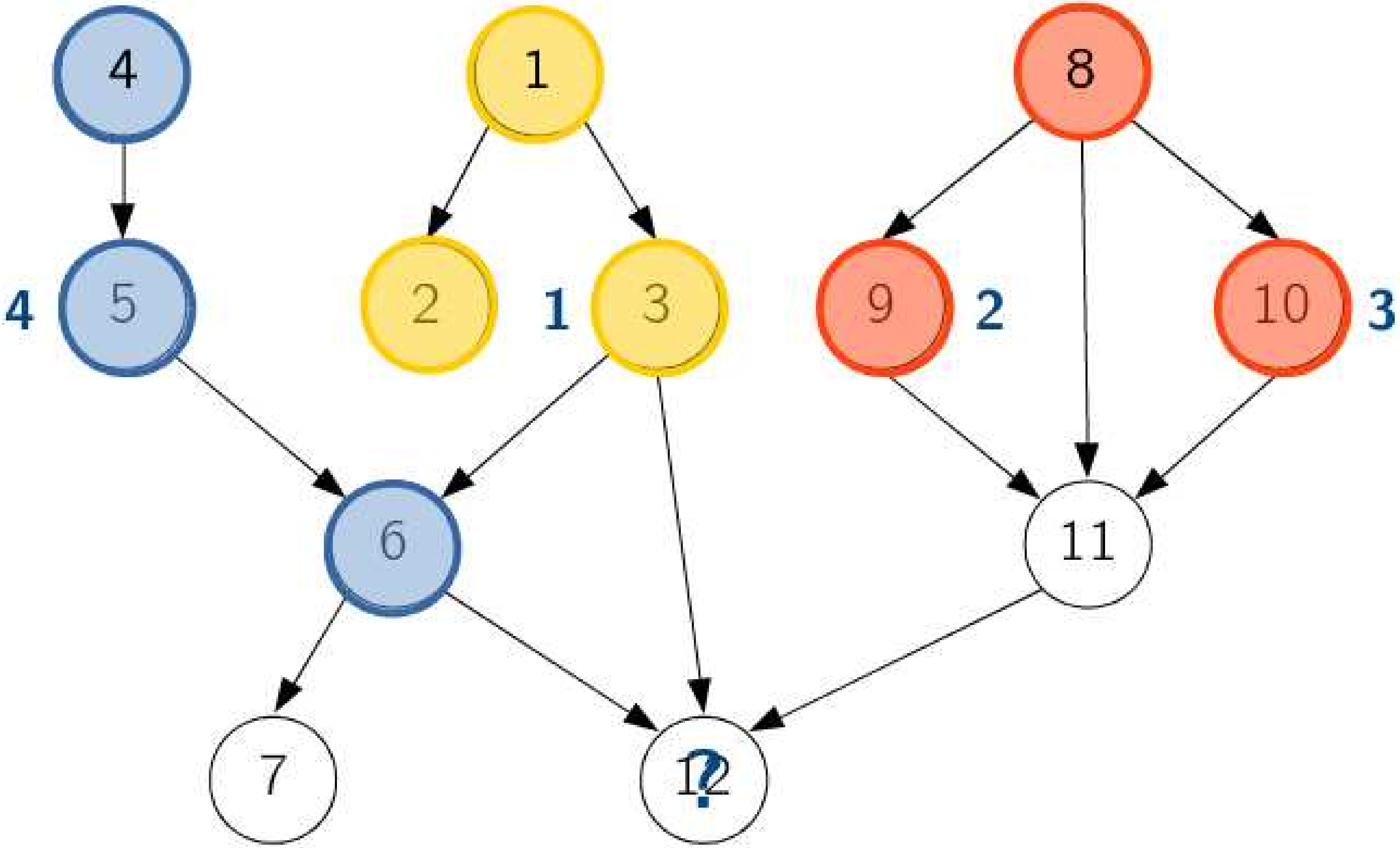

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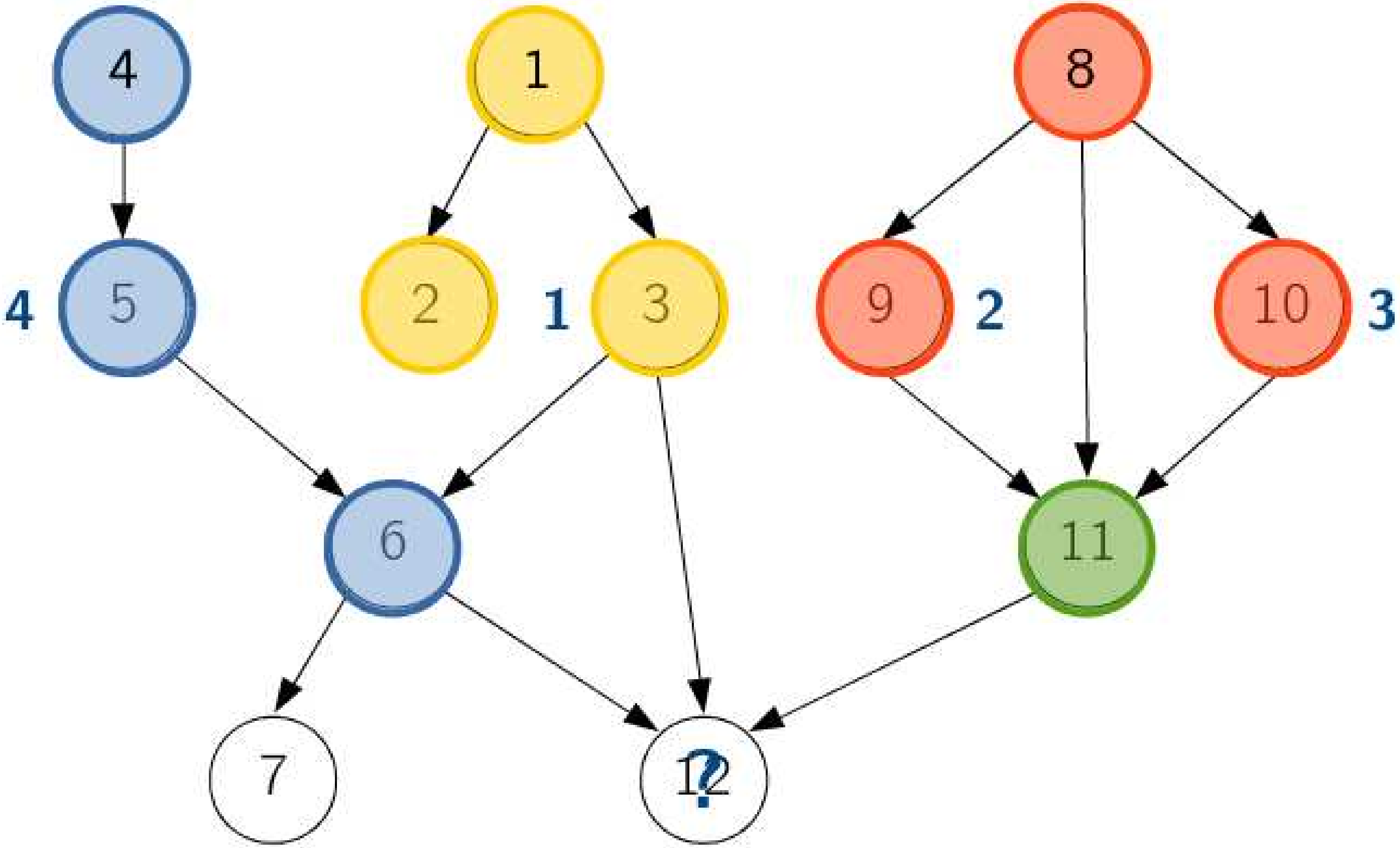

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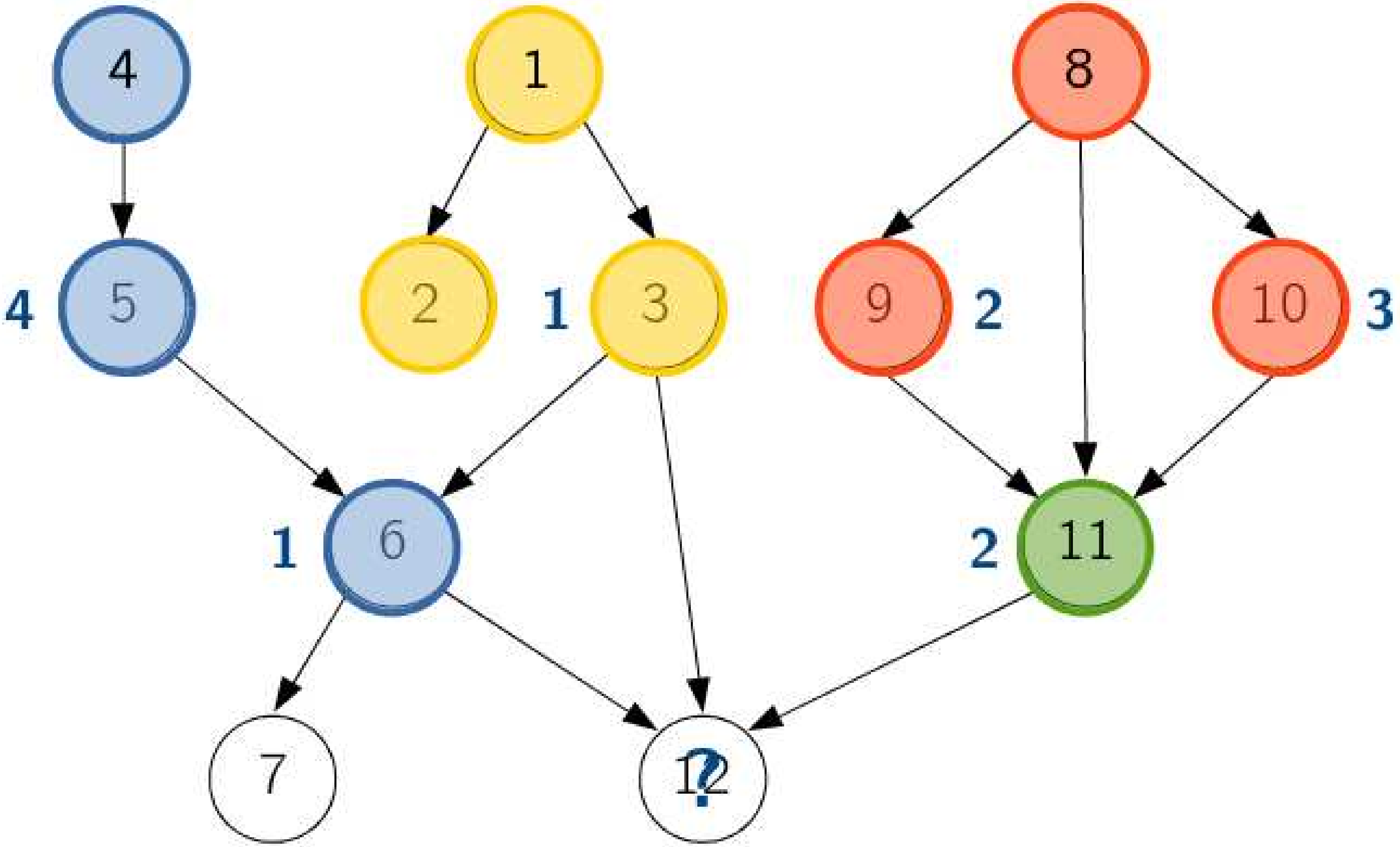

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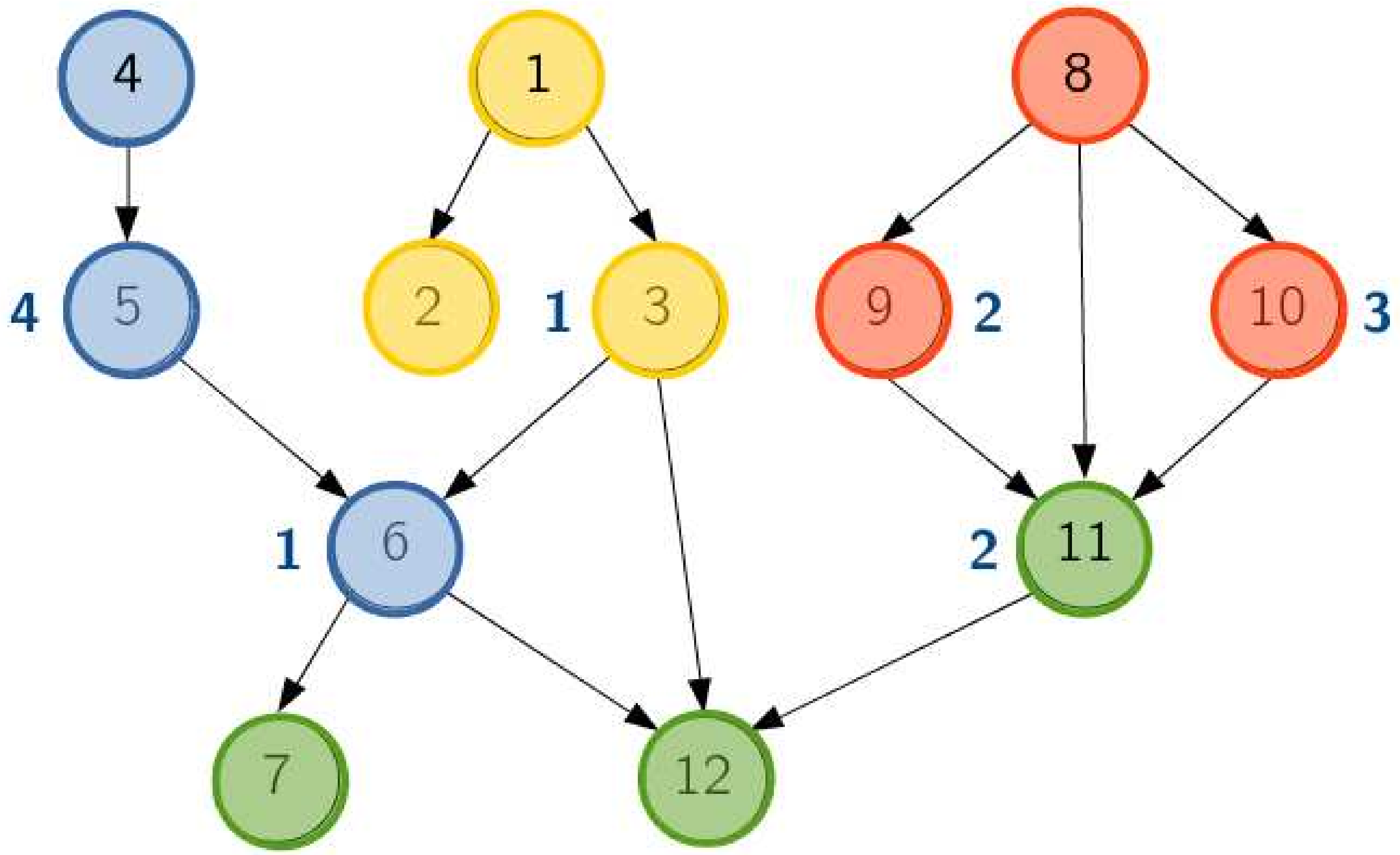

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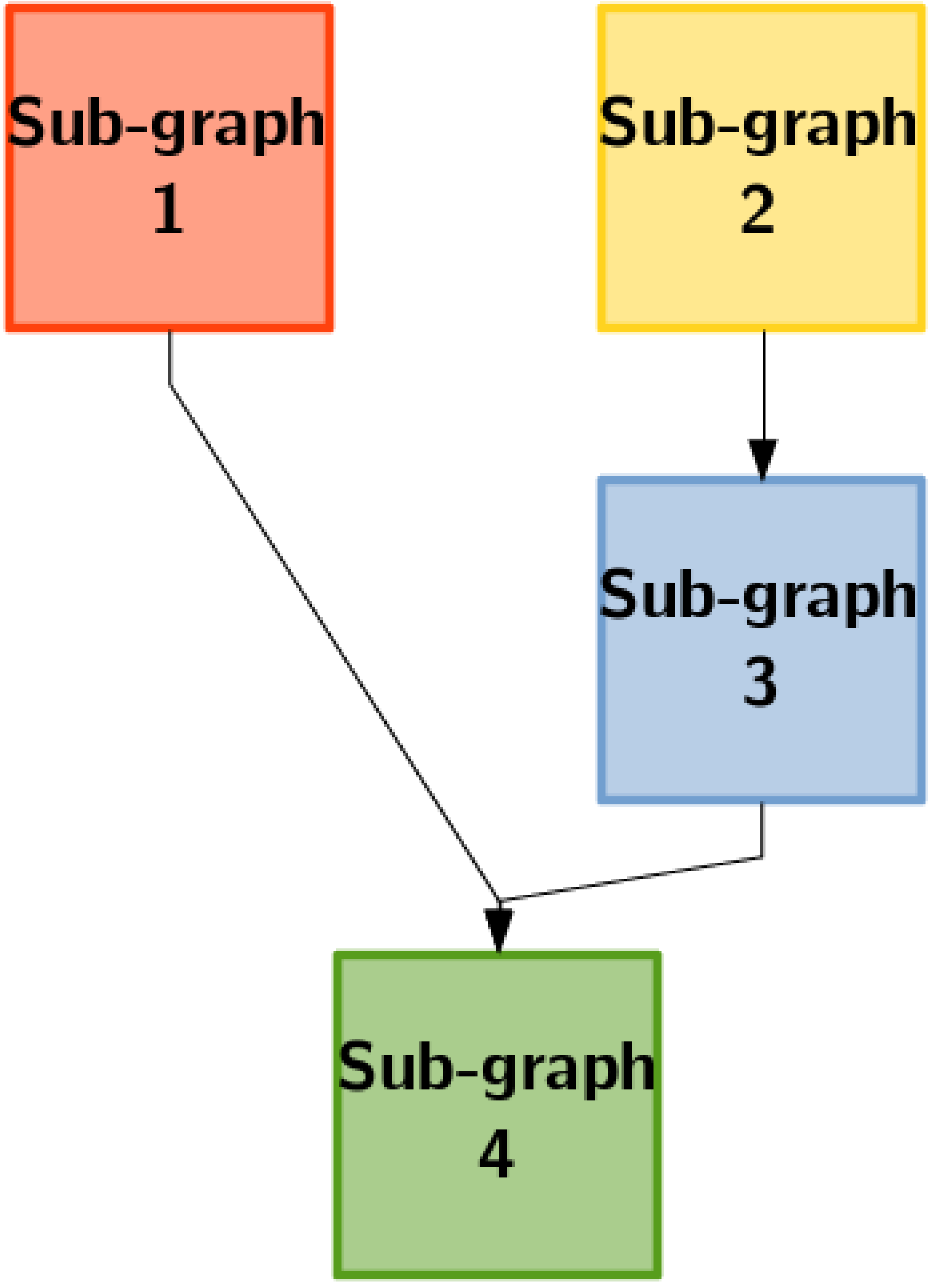
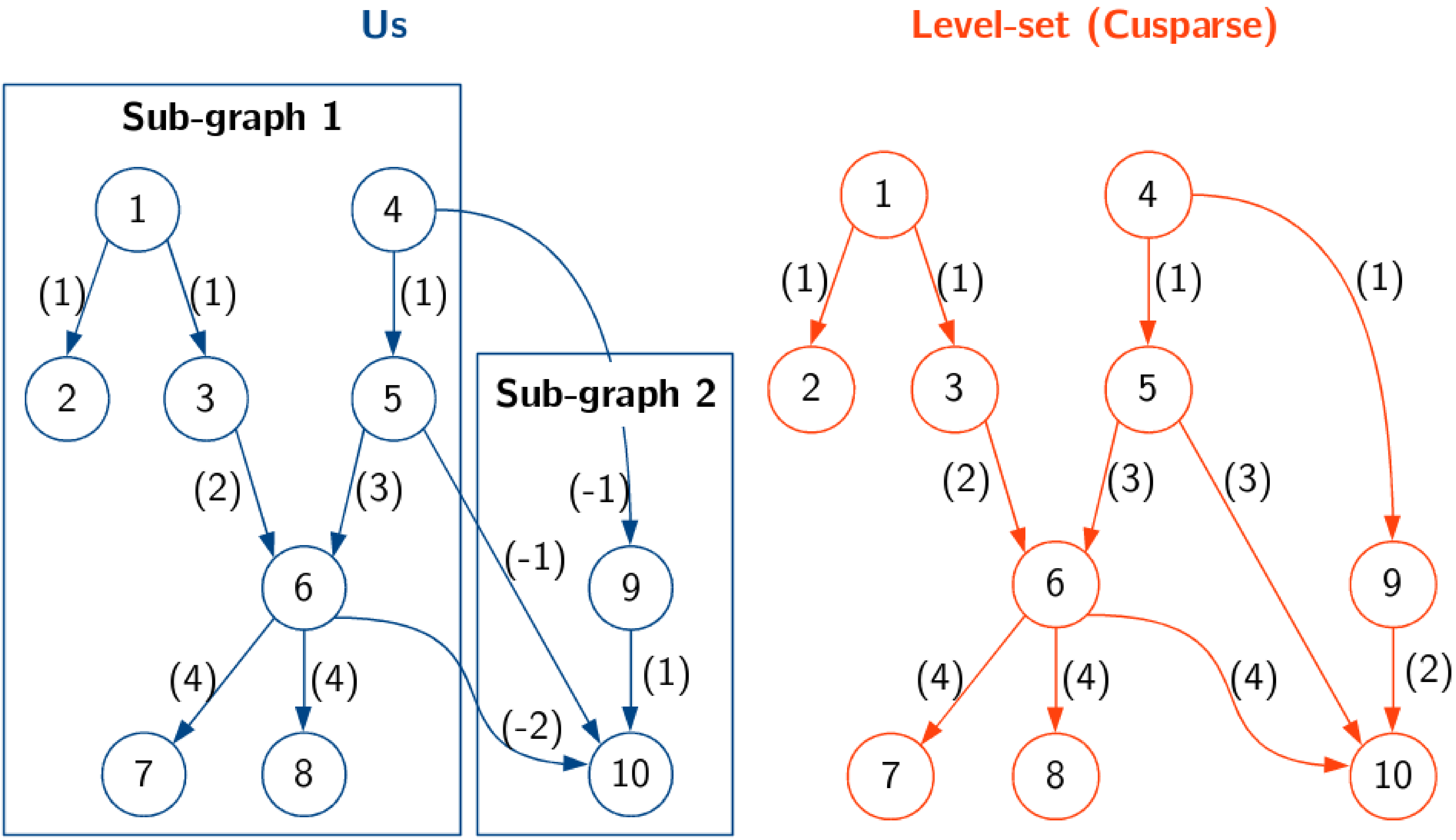


The most used algorithm does not perform any partitioning because it aims to use the whole GPU



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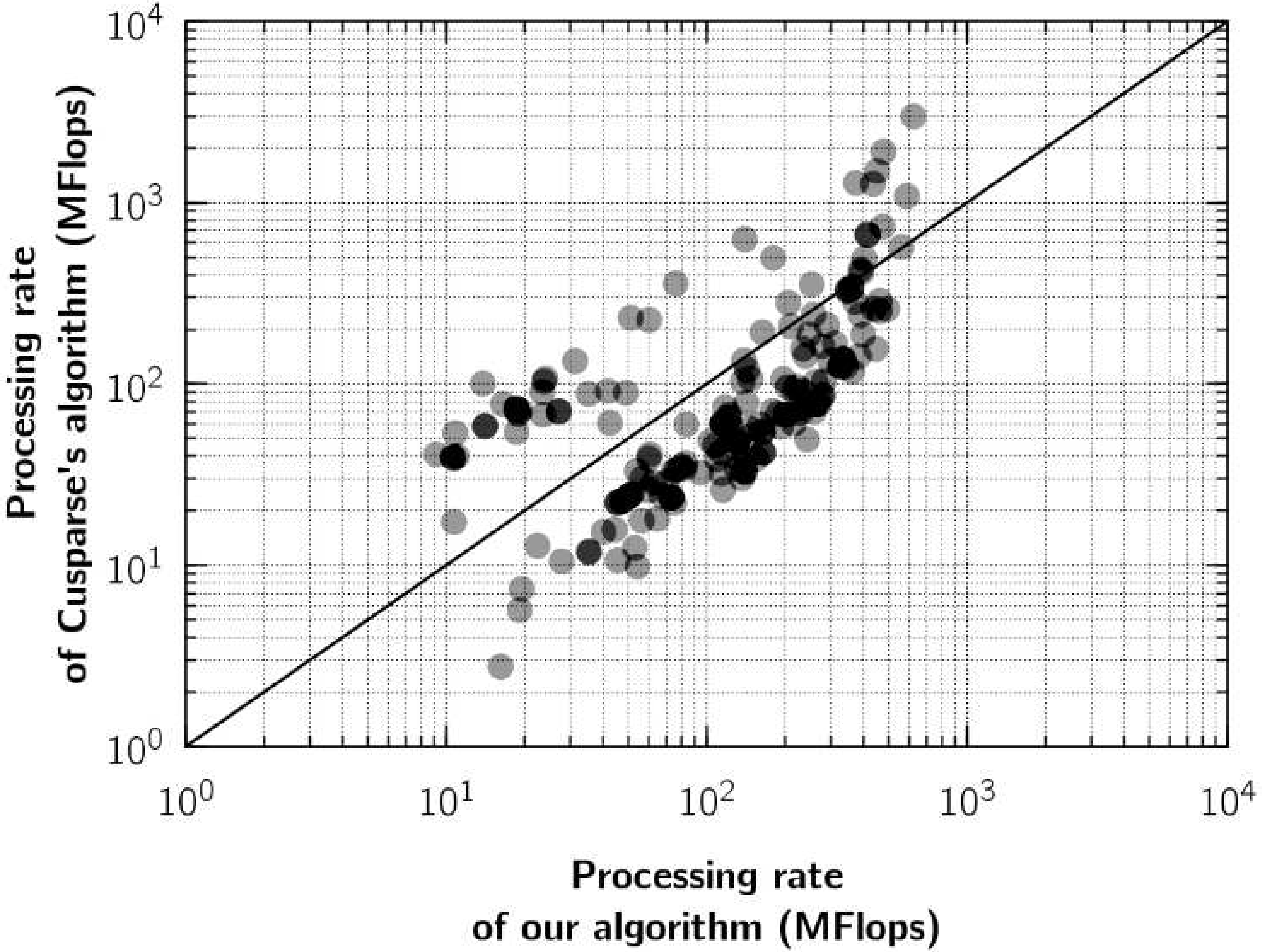
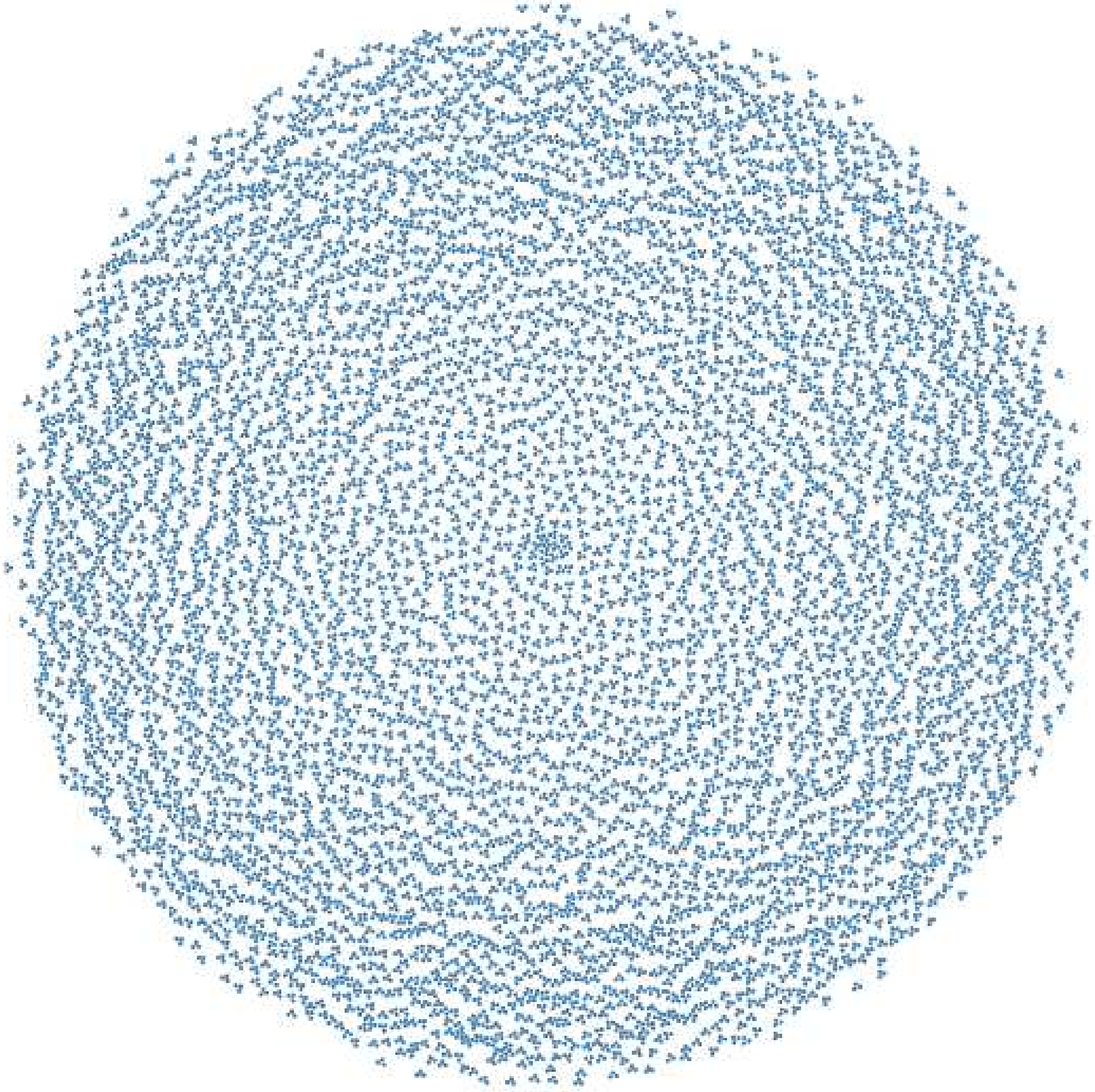
We achieved a maximum speedup of 6x against Cuda's library and a minimum speedup of 0.1x



In the worst case, there was too little data-level parallelism that we could extract



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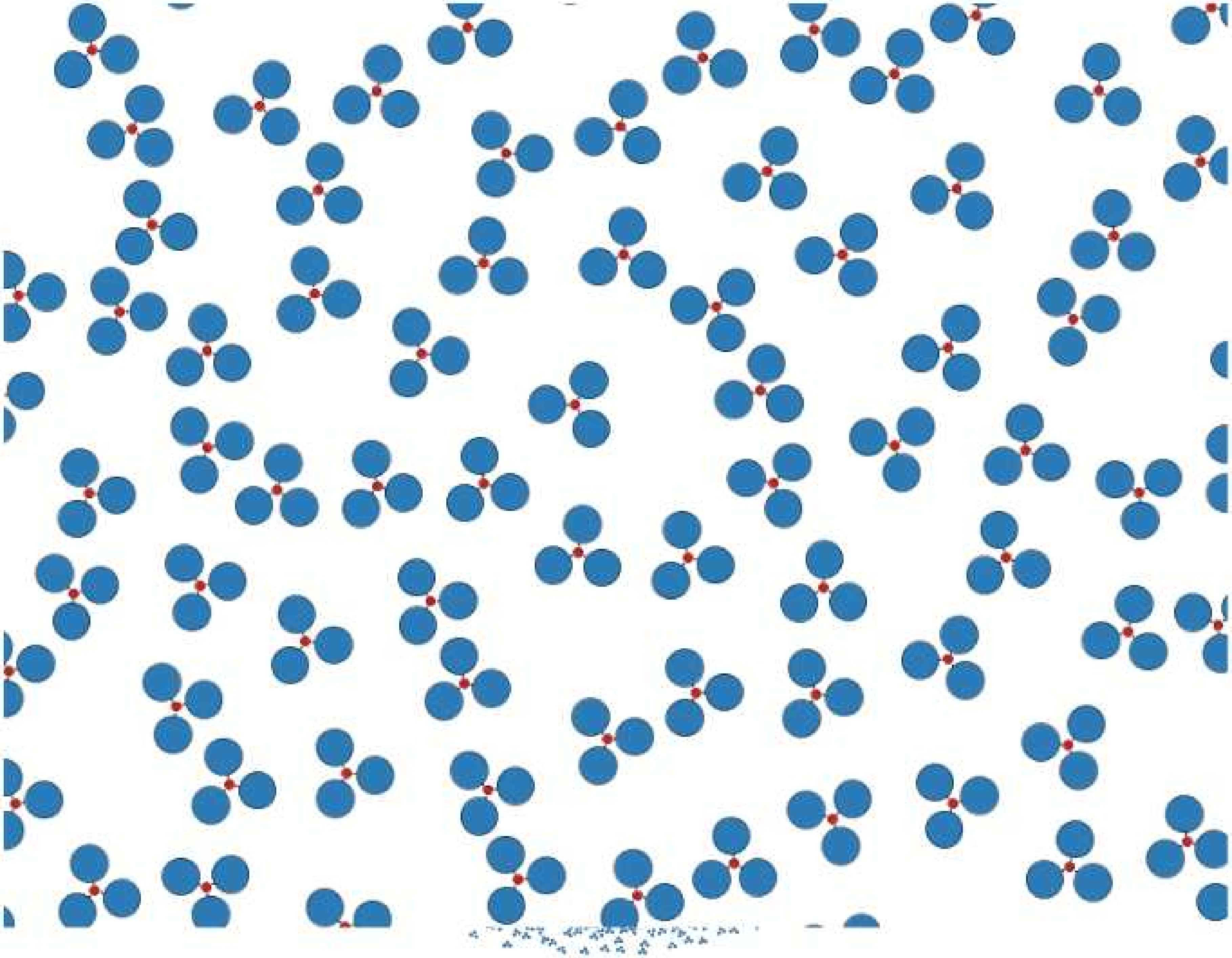
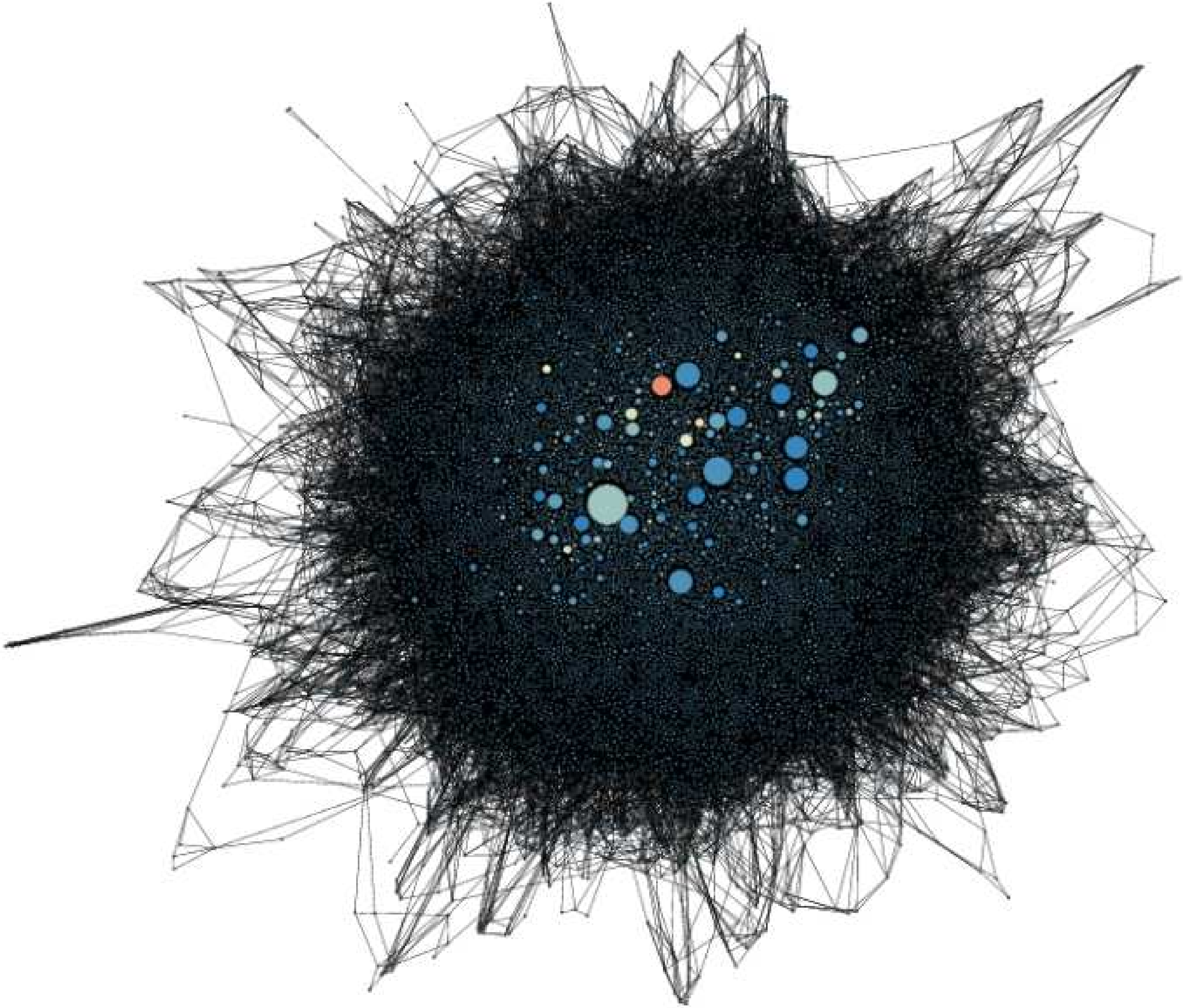


In the best case, we used only one compute unit instead of four, but we were 6 times faster than Cusparse



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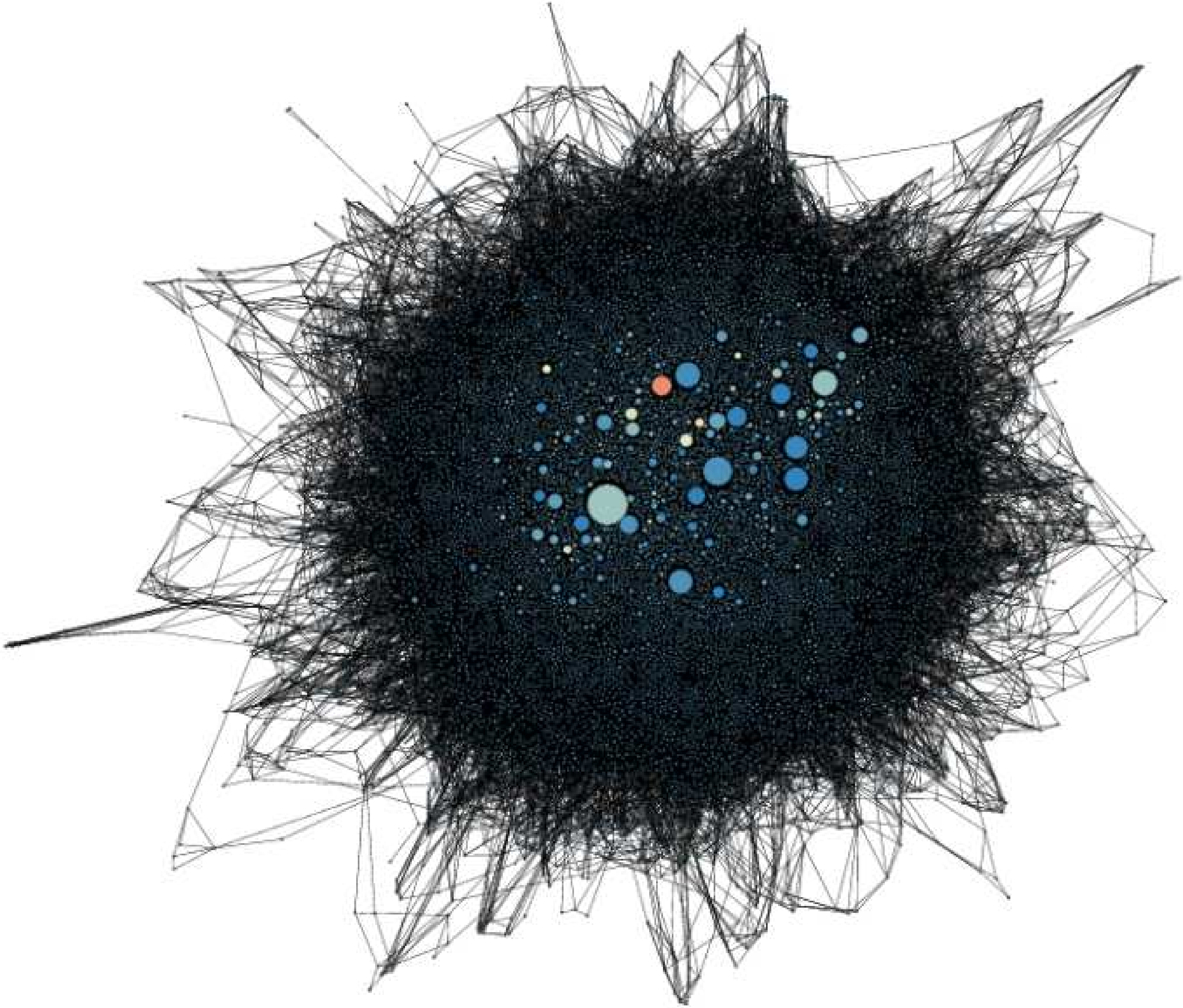
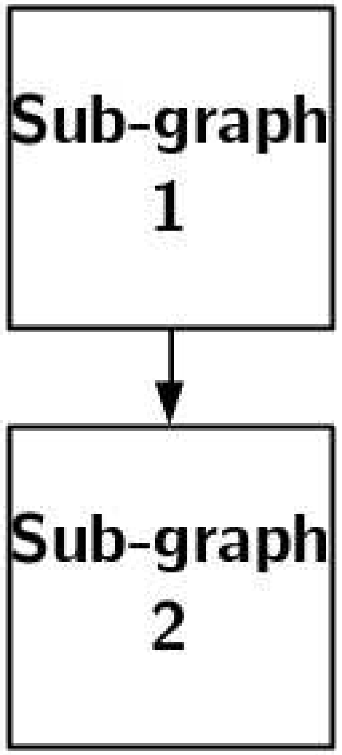
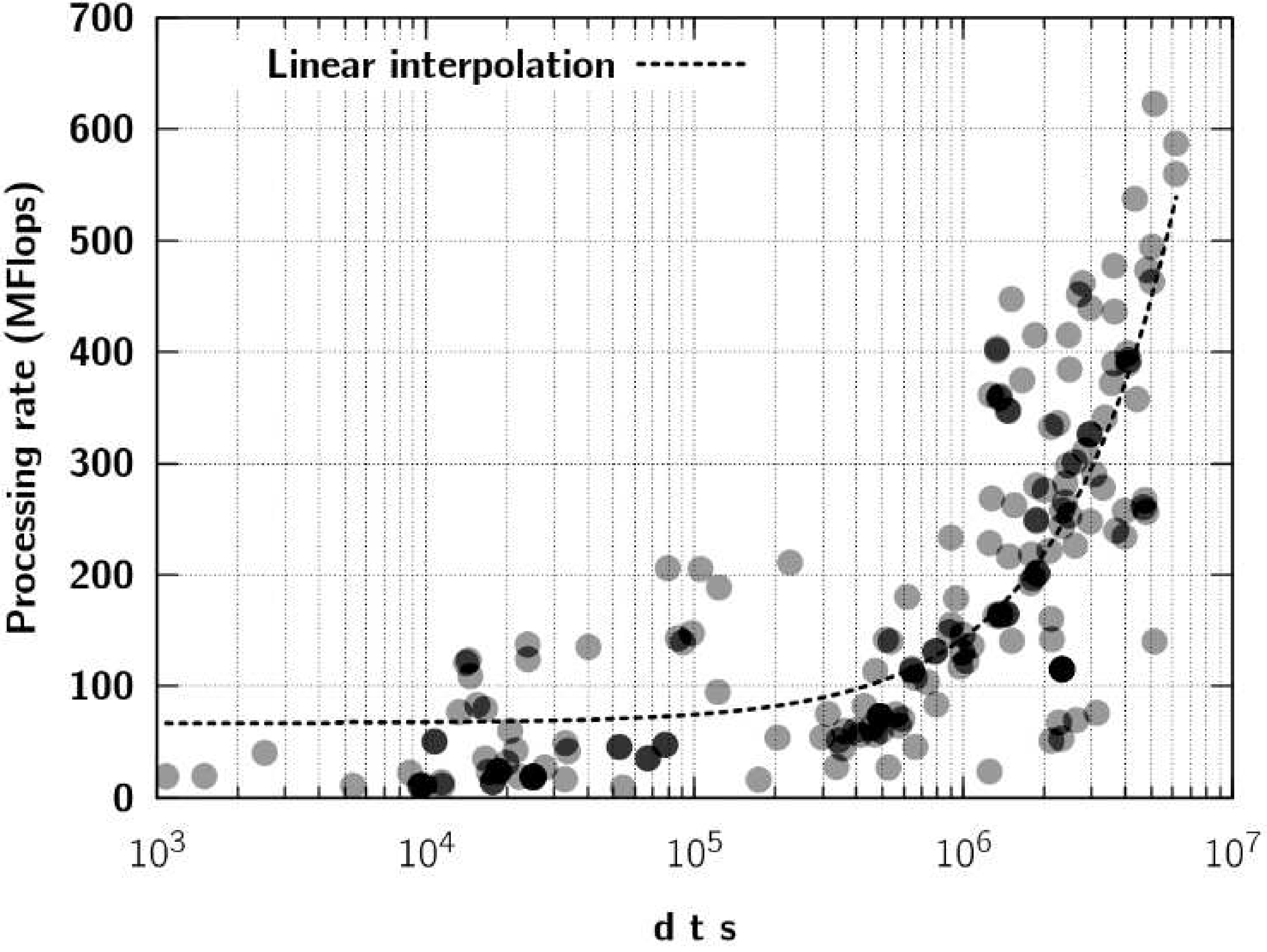




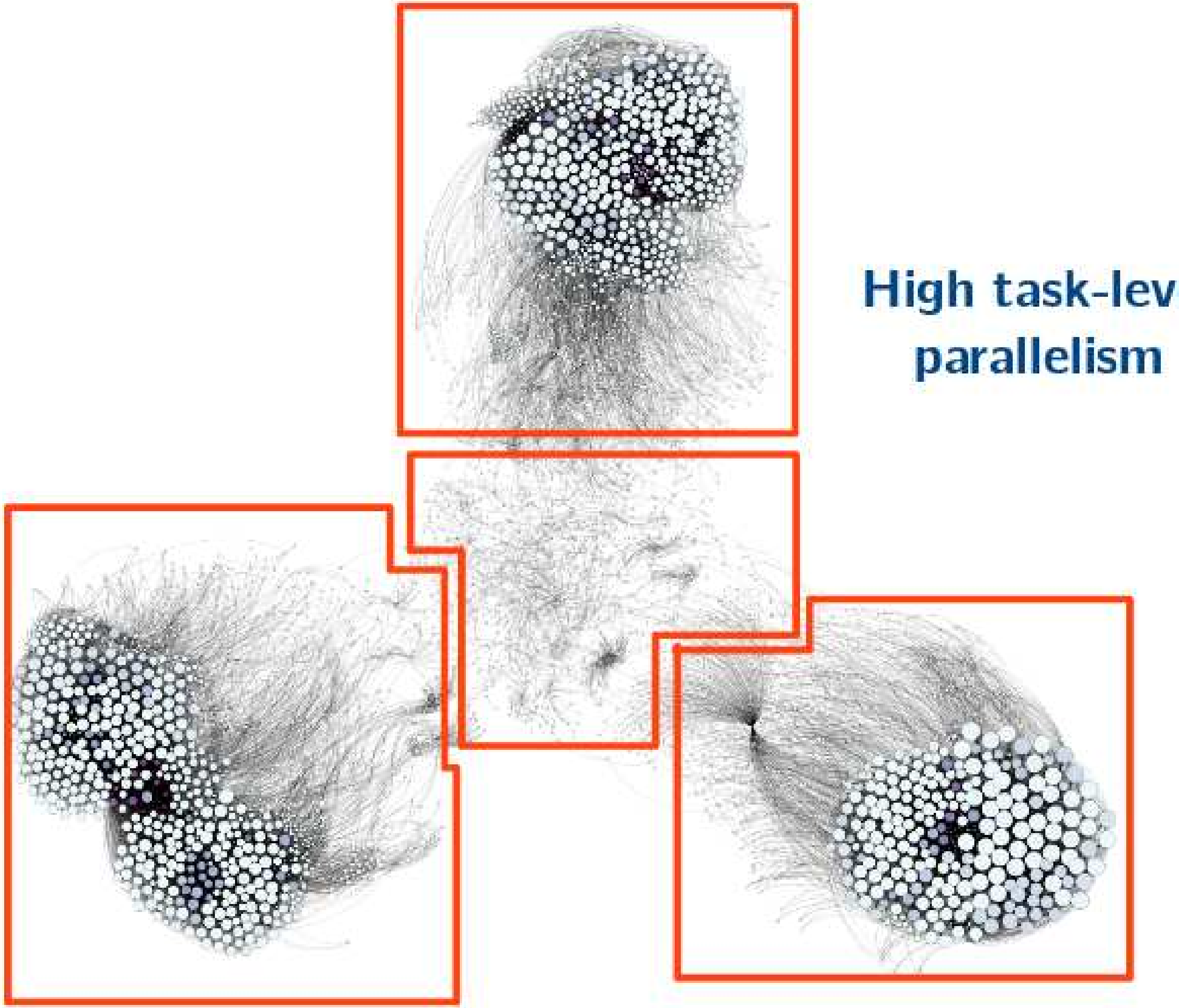
Sub-graph 1

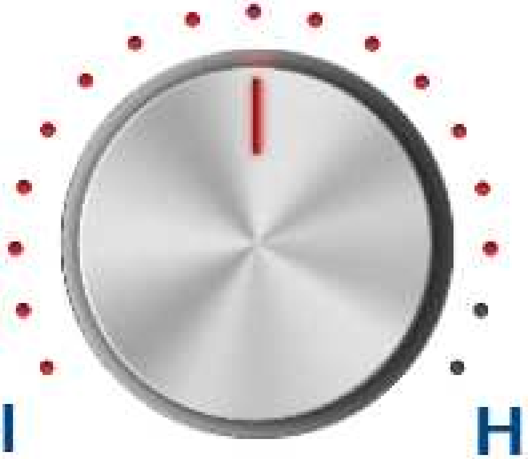
Sub-graph 2

The performance of our algorithm is mainly given by data-level parallelism, task-level parallelism and graph size



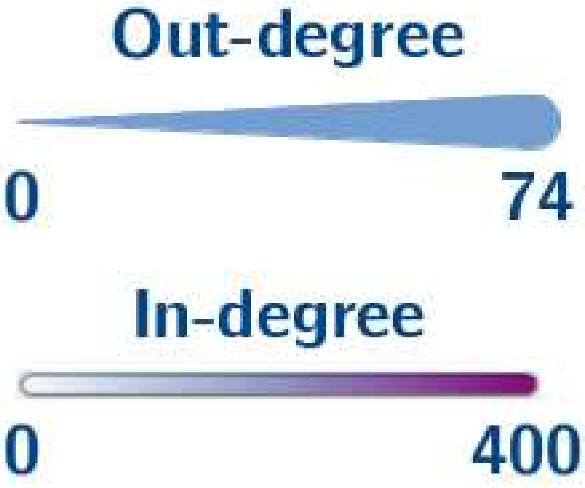
By trading task-level parallelism and data-level parallelism, our algorithm can improve data locality and performance





High task-level parallelism

High data-level parallelism


Out-degree
0
74
In-degree
0
400