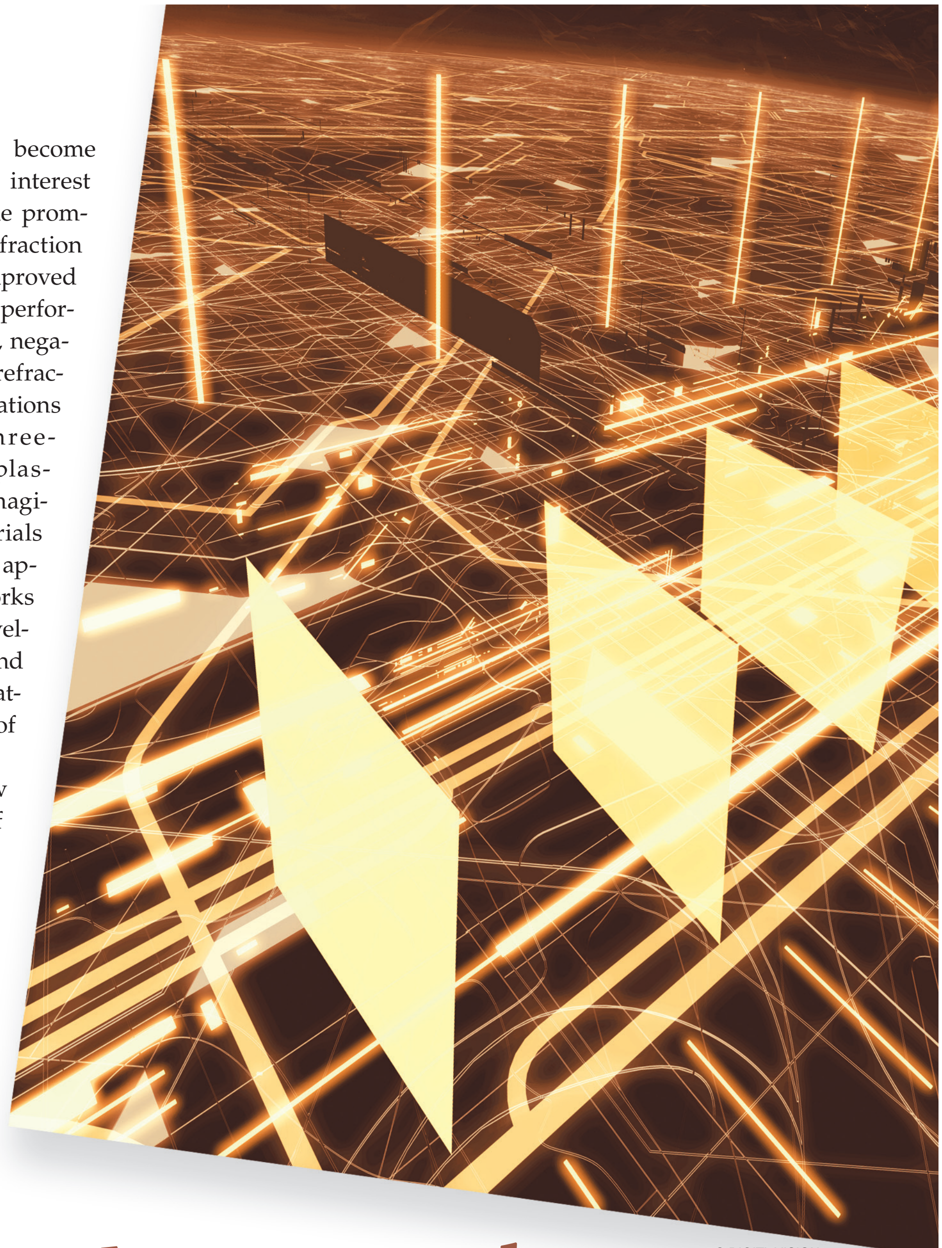
Metamaterials have become a subject of great interest in recent years. The promises of below-diffraction limit focusing, improved stealth, i.e., "invisibility," enhanced performance or design of passive networks, negative permeability, permittivity and refraction, as well as synergistic combinations of two-dimensional (2-D) and three-dimensional (3-D) materials with plasmonic phenomena have fired the imagination of many microwave and materials engineers and scientists. Originally applied to one-dimensional (1-D) networks of passive elements, more recent developments have concentrated on 2-D and 3-D applications and have demonstrated negative refraction in a variety of microwave and optical structures.

In this article, we present a new theory to explain the phenomenon of negative refraction in bulk metamaterials. Unlike earlier theories, it does not rely on the existence of a single mode of propagation in materials with negative constitutive parameters. It is based on the interaction and phase reversal effects caused by the existence of two simultaneous electromagnetic (EM) modes in uniform inhomogeneous



Propagation and Negative Refraction

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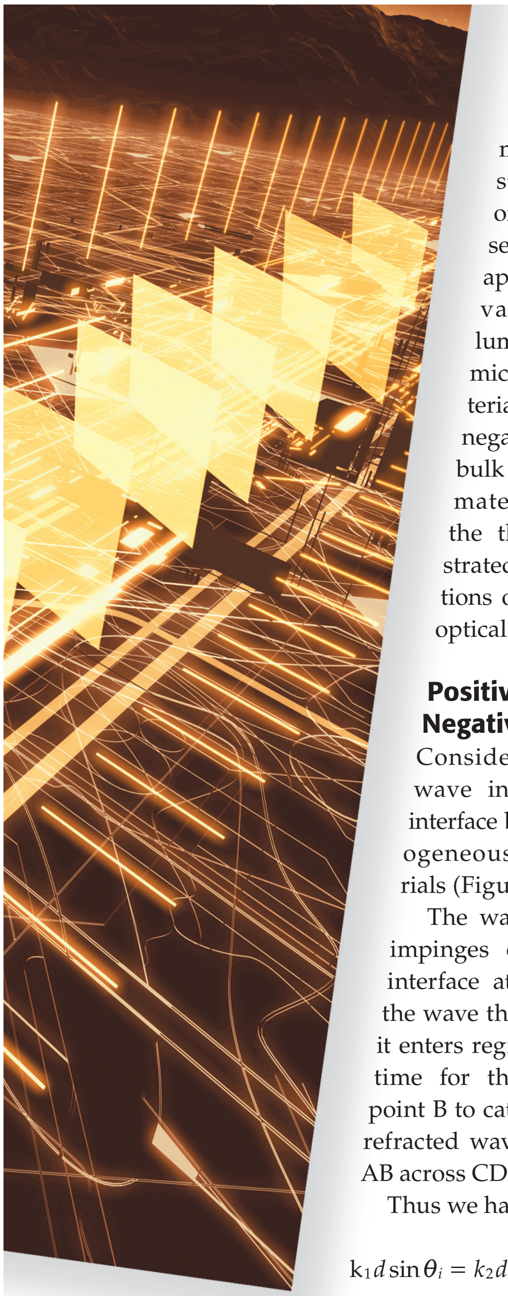

metamaterial structures. The theory is general in the sense that it may be applied with equal validity to quasi-lumped circuit-based microwave metamaterials and to optical negative refraction in bulk nanowire metamaterials. Validity of the theory is demonstrated with EM simulations of microwave and optical metamaterials.

Positive and Negative Refraction

Consider a plane EM wave incident upon the interface between two homogeneous dielectric materials (Figure 1).

The wave front AB first impinges on the dielectric interface at point A, where the wave then slows down as it enters region 2. This allows time for the wave front at point B to catch up, and a new refracted wave front replicates AB across CD.

Thus we have Snell's law:

$$k_1 d \sin\theta_i = k_2 d \sin\theta_r, \tag{1}$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{k_2}{k_1} = \frac{v_1}{v_2} = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}, \tag{2}$$

where $k_{1,2}$ are the wave numbers in the two regions.

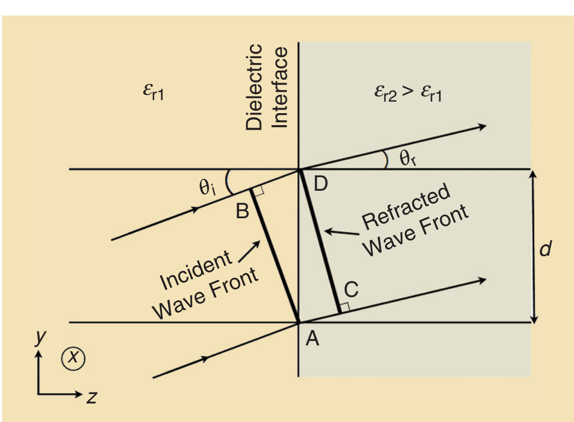

Figure 1. *Positive refraction at a dielectric interface.*

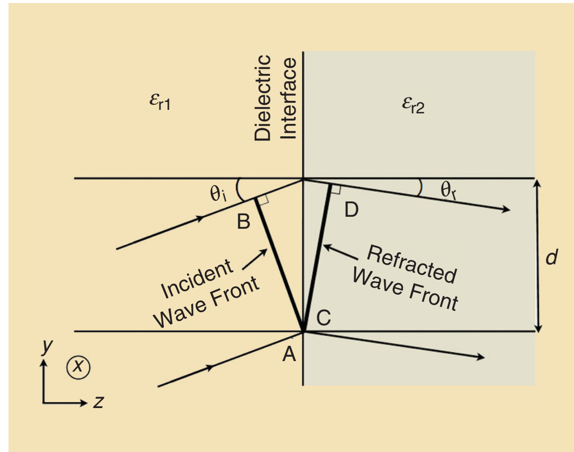

Figure 2. *Negative refraction at a dielectric interface.*

The wave exhibits positive refraction, i.e., the refracted wave is directed closer to the normal to the dielectric interface.

Now consider the possibility of negative refraction as shown in Figure 2.

It is clear that the wave-front AB cannot be replicated at CD as this would require an instantaneous transfer of the field from point B to point D. Consequently, negative refraction at an interface is clearly impossible. If an image, rather than a simple phase front, were to be imposed upon the incident wave, both amplitude and phase information are contained, and the image would then be required to move instantaneously. Of course, information cannot not travel faster than light.

However, all of the papers that discuss negative refraction [1]–[5] describe structures that have finite physical thickness, as shown in Figure 3.

In this case, all that is required from the metamaterial is that, within its thickness t, the wave arriving from point A is slowed down relative to the wave arriving from point B. Thus the incident wave front AB is replicated at CD, and negative refraction can then be said to have occurred.

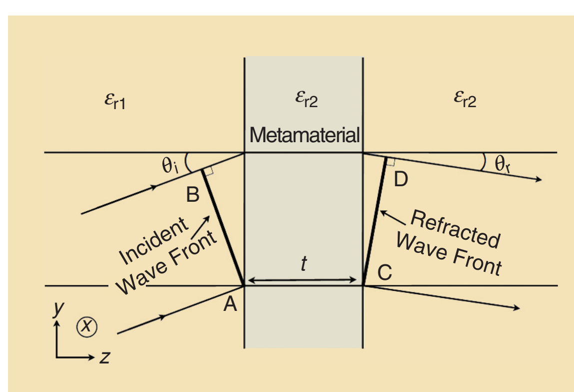

Figure 3. *Negative refraction through a finite thickness metamaterial.*

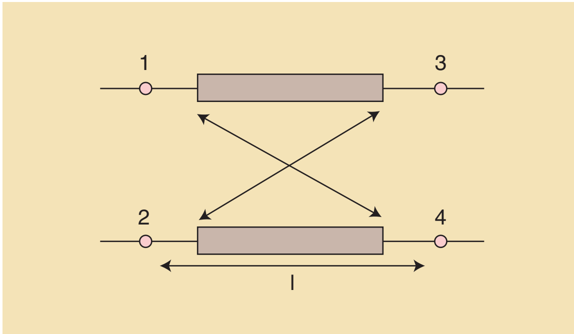

Figure 4. *Inhomogeneous parallel coupled transmission lines.*

One simple example of such a structure is a pair of inhomogeneous parallel coupled transmission lines forming the four-port network shown in Figure 4. Here, the transmission lines have even and odd-mode characteristic impedances Z_{oe} and Z_{oo} and propagation constants β_e, β_o.

From [6] we have

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} \frac{Te+To}{2} & \frac{Te-To}{2} \\ \frac{Te-To}{2} & \frac{Te+To}{2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \tag{3}$$

where E_{1-4} are the fields at ports 1–4.

T_e and T_o are the transmission coefficients of the even and odd modes, where

$$T_e = |T_e|\, e^{-j\beta_e l}, \tag{4}$$

$$T_o = |T_e|\, e^{-j\beta_o l}. \tag{5}$$

Now assume the network is matched at all four ports, i.e., $|T_e| = |T_o|$ and $Z_{oe}.Z_{oo} = 1$. Furthermore impose the condition

$$(\beta_e - \beta_o) l = \pi. \tag{6}$$

Thus

$$\frac{T_e + T_o}{2} = 0, \tag{7}$$

$$\frac{T_e - T_o}{2} = |T_e|\, e - {}^{j\beta_e l}. \tag{8}$$

Hence

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} 0 & |T_e|e^{-j\beta_e 1} \\ |T_e|e^{-j\beta_e 1} & \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}. \tag{9}$$

Thus the waves at ports 3 and 4 are reversed relative to those at ports 1 and 2, and the incoming wave at ports 1 and 2 would experience negative refraction.

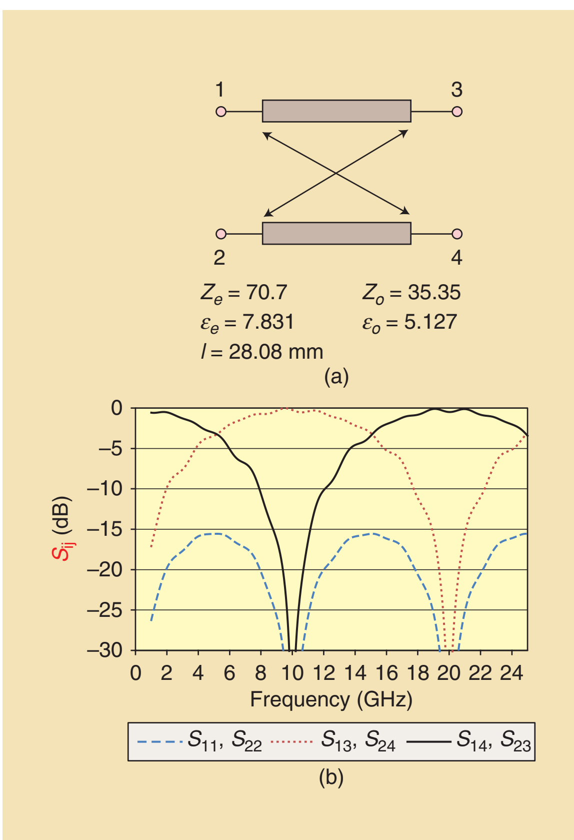

Figure 5. *(a) Microstrip parallel coupled line and (b) its circuit simulation.*

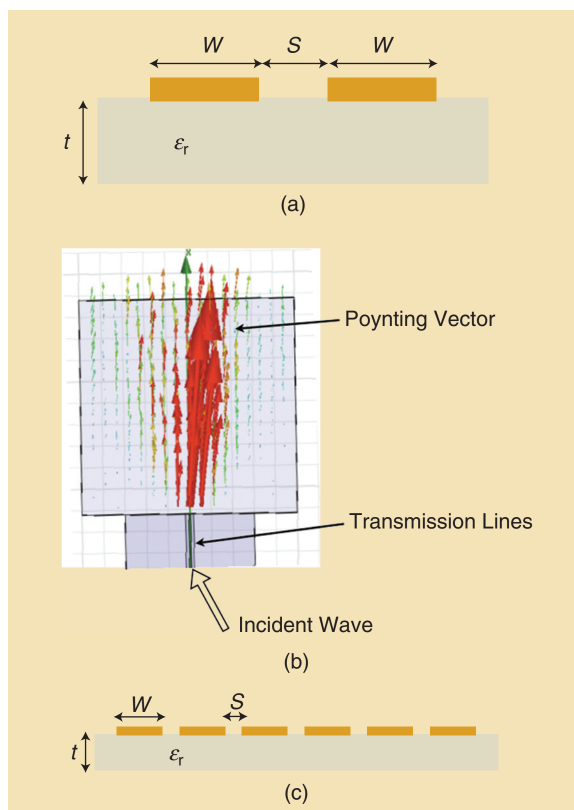

Figure 6. *(a) Field simulation of the coupled microstrip lines structure with an incident wave at 45° (w = 1 mm, s = 0.5 mm, t = 1 mm) and (b) an extension to multiple coupled lines.*

In order to demonstrate this principle, a matched inhomogeneous two-wire microstrip coupled line was designed with its length chosen such that its even and odd-mode phase lengths differed by 180° at 10 GHz [Figure 5(a)]. Circuit simulations of this network are shown in Figure 5(b). It was observed that, at 10 GHz, S_{13} and S_{24} were unity and S_{14} and S_{23} were zero. Doubling the effective length of the structure has the effect of making the phase difference between the even and odd modes equal to 360°, which results in S13 and S24 being zero. This may be observed in the response at 20 GHz.

The cross-sectional dimensions of the network are shown in Figure 6(a). Note that this structure can be extended to the multiple coupled lines shown in Figure 6(b). Field simulation of the physical structure with an incident wave at 45° was performed using HFSS. The results clearly demonstrate negative refraction. However, the output wave was not particularly well focused, and this will be discussed later in this article.

Structures with 2-D Cross Section

In the previous section, it was demonstrated that negative refraction cannot occur across an interface but requires a finite thickness metamaterial. Furthermore, it has been demonstrated that phase cancellation between the even and odd propagating modes in a four-port inhomogeneous coupled line structure can achieve this objective. This basic theory will now be extended to structures with a truly 2-D cross section. (Note that in [1]–[5] it is assumed that the metamaterials have only a single mode of propagation, however, it is shown in [7] that uniform inhomogeneous structures will, in general, always support more than one mode.)

First we consider the lumped network with 2-D cross section shown in Figure 7. This is similar in principle to the networks described in [1].

It is assumed that all the elements are small with respect to the wavelength at the frequency of excitation and the structure extends to infinity in the xy plane. It is of finite thickness l, in the z direction.

Now consider a plane wave incident at an angle θ_i to the front surface (xy plane) of the structure.

The incoming plane wave is linearly polarized with the electric field directed along the x axis and the magnetic field directed along the incident wave front. The wave will excite transverse electric (TE) waves within the lumped metamaterial. Alternatively, if the electric and magnetic fields are interchanged so that the magnetic field points along the x axis, then the wave will excite transverse magnetic (TM) waves within the metamaterial (Figure 8). (Note that TE waves have purely transverse components of electric field, i.e., no electric field directed along the z-axis. Similarly, TM waves have purely transverse components of magnetic field, etc.)

If the structure is assumed to be infinite in the xy plane, then it behaves the same at all points in that plane. Therefore, in the TE case, we can place perfect electric conductors (PECs) along lines of symmetry

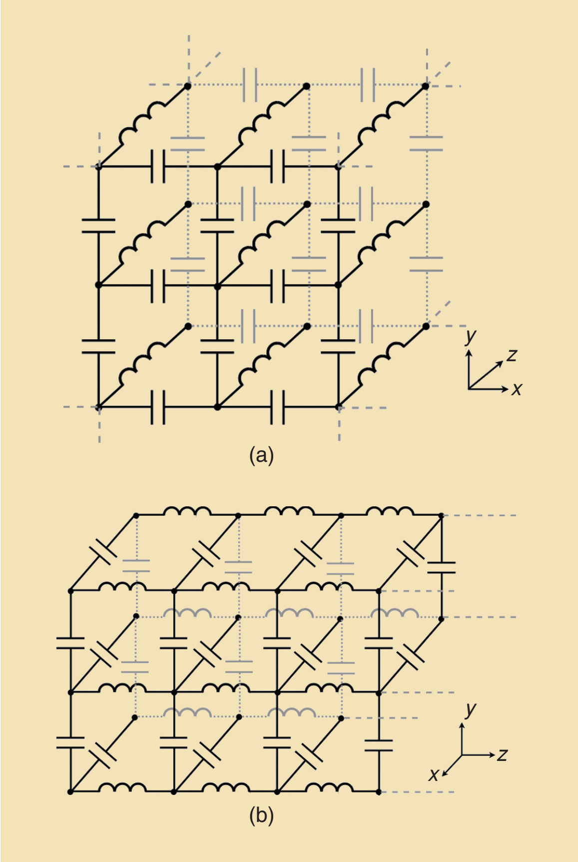

Figure 7. *2-D lumped metamaterial.*

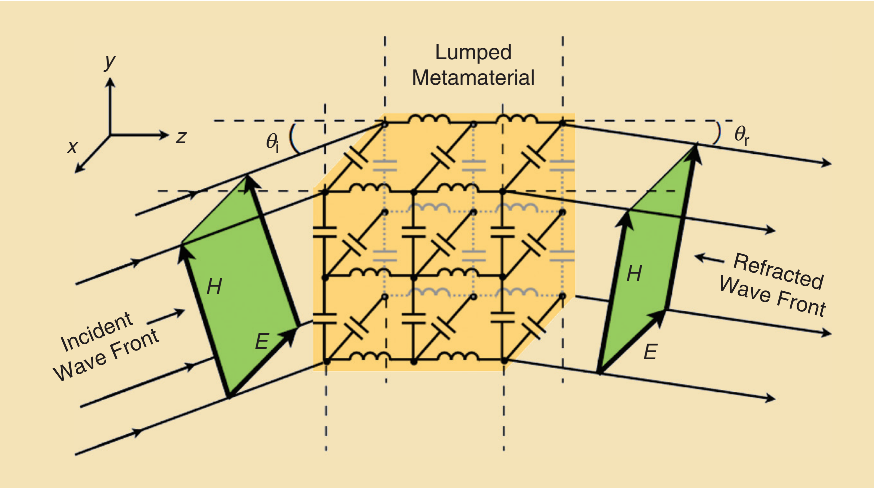

Figure 8. *Lumped metamaterial—TE excitation.*

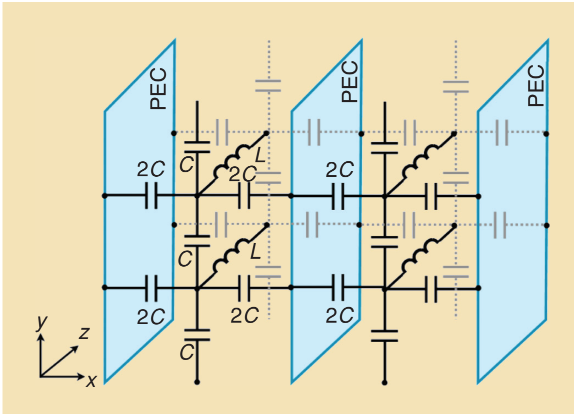

Figure 9. *TE excitation of metamaterial.*

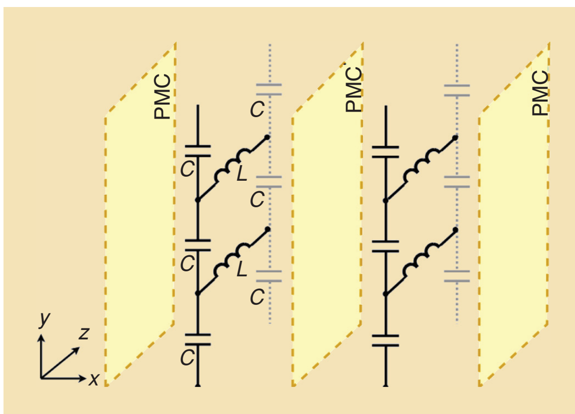

Figure 10. *TM excitation of metamaterial.*

(y-axis) between the nodes at which the inductors are connected without altering the field pattern (Figure 9). (The electric field tangential to the PEC is zero.) Similarly, in the TM case, we can insert perfect magnetic conductors (PMC) (Figure 10). (The magnetic field tangential to the PMC is zero.) Therefore, it may be described in terms of the even and odd modes of the network for either the TE or TM waves. Note that an even mode excitation is equal potential on adjacent input nodes, hence there is an open circuit (PEC) along the plane of symmetry. Likewise, odd-mode excitation is opposite potential on adjacent input nodes, therefore a short circuit (PMC) along the plane of symmetry.

For the TE case, the even and odd mode equivalent circuits of the structure are shown in Figure 11.

Note that there can be no capacitance from a node to a magnetic wall. The propagation constant for a transmission line is defined as

$$\beta = \omega\sqrt{L_s C_{sh}}\,, \tag{10}$$

where L_S is the series inductance and C_{Sh} is the shunt capacitance.

For the $\mathrm{TE_e}$ case

$$L_S = L \text{ and } C_{Sh} = 4C \quad \text{then} \quad \beta_{Ee} = 2\omega\sqrt{LC}\,. \tag{11}$$

For the $\mathrm{TE_o}$ case

$$L_S = L \text{ and } C_{Sh} = 8C \quad \text{then} \quad \beta_{Eo} = 2\sqrt{2}\,\omega\sqrt{LC}\,. \tag{12}$$

The even and odd modes have different phase velocities, and arranging for $(\beta_e - \beta_o)l = \pi$ will result in negative refraction at the output face of the structure.

The TM mode behaves differently. The even and odd mode equivalent circuits are shown in Figure 12. Here we see that the propagation constant of the $\mathrm{TM_o}$ mode is the same as the $\mathrm{TE_e}$ case, i.e.,

$$\beta_{MO} = 2\omega\sqrt{LC}\,. \tag{13}$$

However, there is no capacitance to ground in the $\mathrm{TM_e}$ case, and the structure behaves as a series inductance. This mode is evanescent and will be highly attenuated at the output of the structure. Thus there is only a single propagating mode in the TM case, and therefore negative refraction cannot occur.

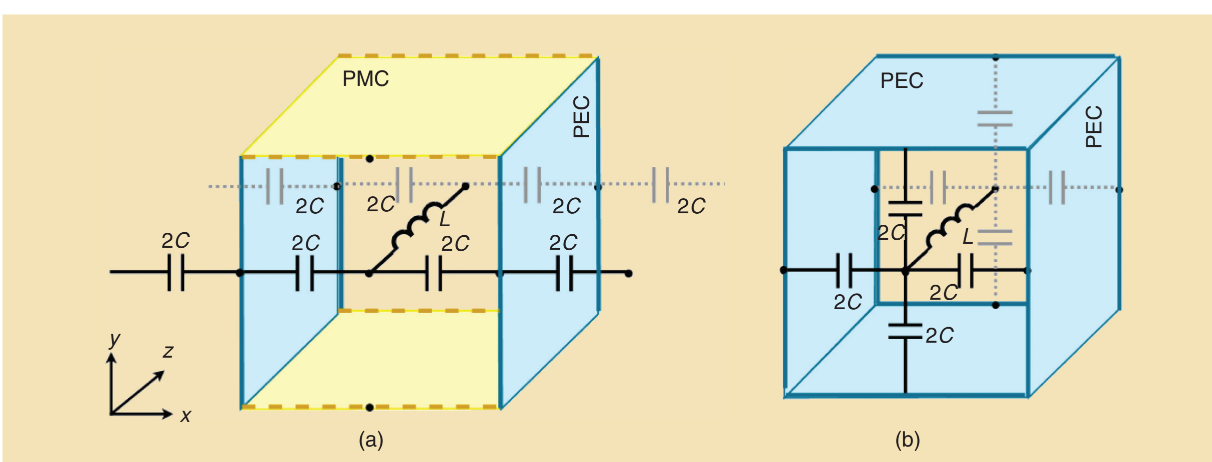

Figure 11. *Equivalent circuits for even and odd modes in the TE case.*

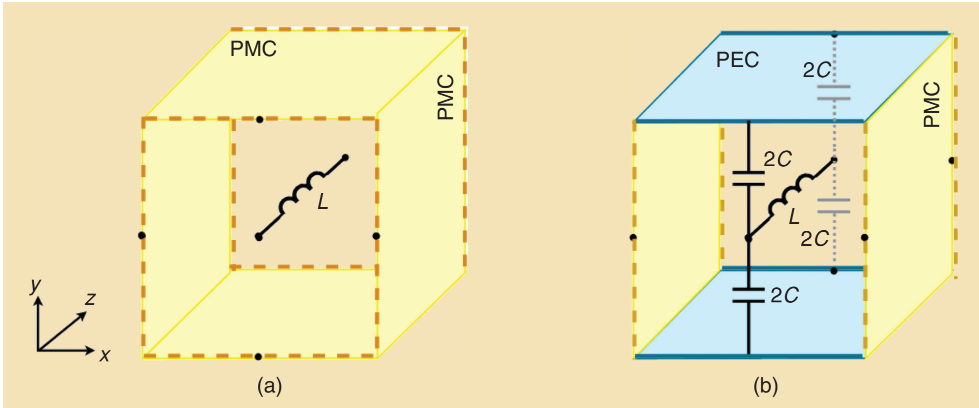

Figure 12. *Equivalent circuits for even and odd modes in the TM case.*

Now consider the case of the optical metamaterial described in [3]. The cross section of the structure is an array of nanowires embedded in porous alumina (Figure 13). In [3], it was reported that this structure exhibited optical negative refraction for wavelengths of 66 nm and that it operated over a broad bandwidth with no observed resonances. Therefore, it is reasonable to neglect any plasmon effects [8] and treat the structure as the optical equivalent of a 2-D array of coupled transmission lines. The basic equivalent circuits for the TE and TM modes are thus as shown in Figure 14.

If the alumina dielectric is assumed to be homogeneous, then the TE_e, TE_o, and TM_o will all be TEM modes with identical propagation constants. The TM_e mode will have no admittance to ground and will be a highly attenuated evanescent mode. Thus, negative refraction cannot occur in either the TE or TM cases.

However, the structure is described as being constructed from porous alumina. Thus, the dielectric will be inhomogeneous and there will be a small difference between the propagation constants of the TE_e and TE_o modes, and, therefore, negative refraction is possible in the TE case.

In order to demonstrate the theory the structure described in [3] was simulated. Initially, a basic cell shown in Figure 15 was modelled.

This was simulated using AnSys High Frequency Structure Simulator (HFSS), and the porosity of the region was modelled by introducing a rectangular air region. The dimensions of this region were adjusted until there was 180° phase difference along the length of the structure between the even and odd modes at a frequency of 450 THz.

The nine-wire optical metamaterial shown in Figure 16 was then simulated for TE excitation using HFSS. This was simulated with and without the air region, and the results are shown in Figure 17.

Here it is seen that the homogeneous structure exhibits positive refraction, and the inhomogeneous structure clearly exhibits negative refraction.

Focusing of Negative Refraction

It has been demonstrated that negative refraction may occur in both lumped and distributed metamaterials of a 2-D cross section due to phase differences between the TE_e and TE_o modes. However, the theory so far has been limited to the four-port network shown in Figure 18,

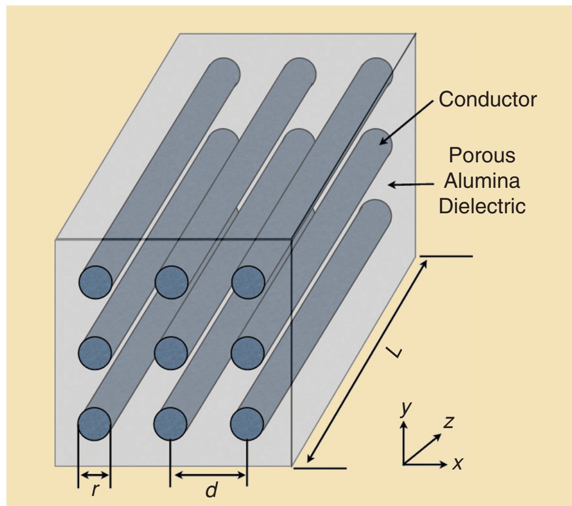

Figure 13. *Optical metamaterial of nanowires (r = 60 nm, d = 110 nm, L = 4.5 μm [3]).*

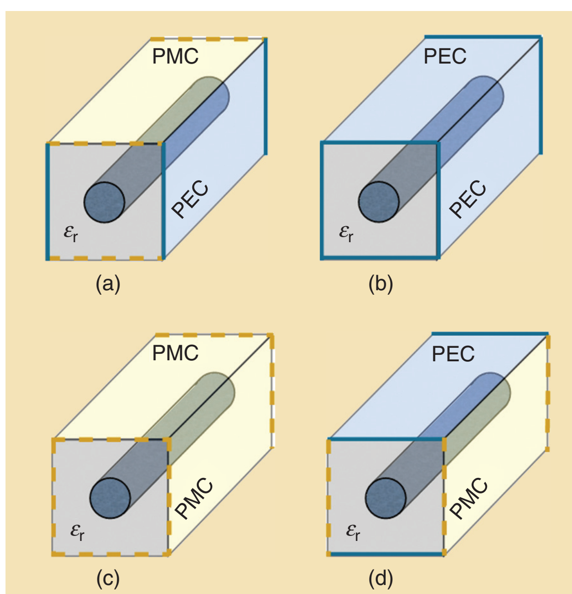

Figure 14. *Equivalent circuits of even and odd modes of optical metamaterial for TE and TM excitation. (a) TE_e, (b) TE_o, (c) TM_e, and (d) TM_o.*

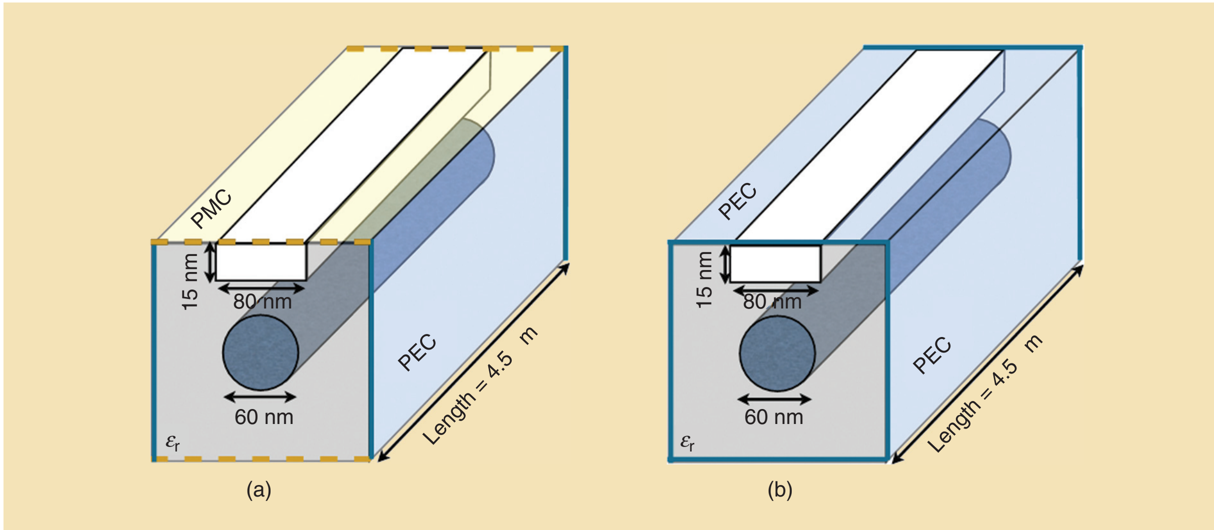

Figure 15. *HFSS model of the porous alumina. (a) TE_e and (b)TE_o.*

which may be considered as a basic building block of the N element case.

The simulated performance of a nine-wire model of the optical metamaterial clearly shows negative refraction. However, the results do not show clear focusing at the output. We will now consider a connection of multiple four-port building blocks with an incident wave at an angle θ_i as shown in Figure 19.

Here we consider the yz plane only as the fields are constant along the x axis. The spacing between elements d is assumed to be small relative to the wavelength.

As the incident wave hits the front surface of the metamaterial, there is a gradually increasing phase lag along the y axis, increasing by increments of φ between each element, where

$$\varphi = kd \sin\theta = \frac{2\pi}{\lambda} d \sin\theta. \tag{14}$$

The input phase lag must be compensated for in order to achieve proper focusing at the output. It should be noticed that in the optical N-wire line reported in [3], the input wave is excited through a thin slit and this is not an issue. An alternative method is to achieve correct focusing by extending the output surface to form a prism so that each basic section has an extra phase shift in a similar way to that reported in [9].

The fields at each output node consist of two components, one from each of its adjacent input nodes, and the output fields are

$$E_1 = e^{-j6\varphi} + e^{-j(4\varphi+\Phi)}, \tag{15}$$
$$E_2 = e^{-j(5\varphi+\Phi)} + e^{-j(3\varphi+2\Phi)}, \tag{16}$$
$$E_3 = e^{-j(4\varphi+2\Phi)} + e^{-j(2\varphi+3\Phi)}, \tag{17}$$

etc.

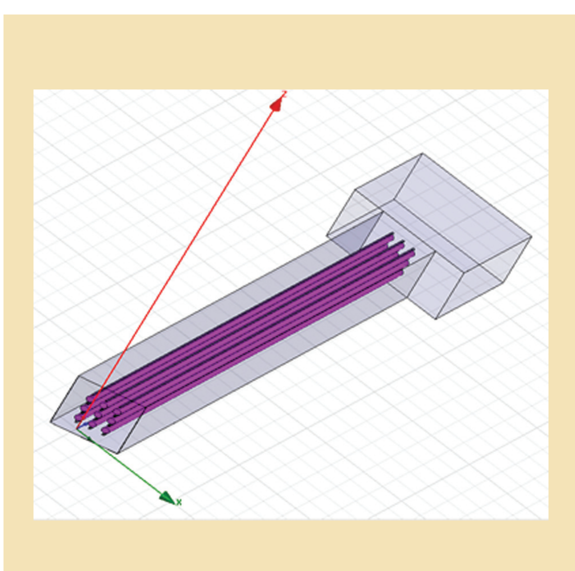

Figure 16. *Nine-wire optical metamaterial.*

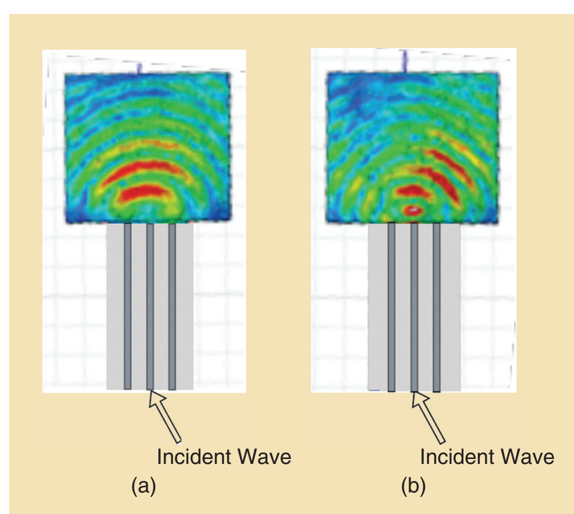

Figure 17. *HFSS simulation with TE excitation of nine-wire optical metamaterial: (a) homogeneous and (b) inhomogeneous.*

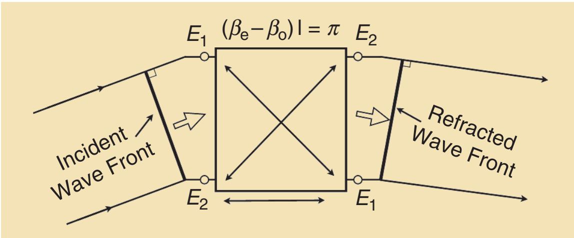

Figure 18. *Four-port building block for metamaterials.*

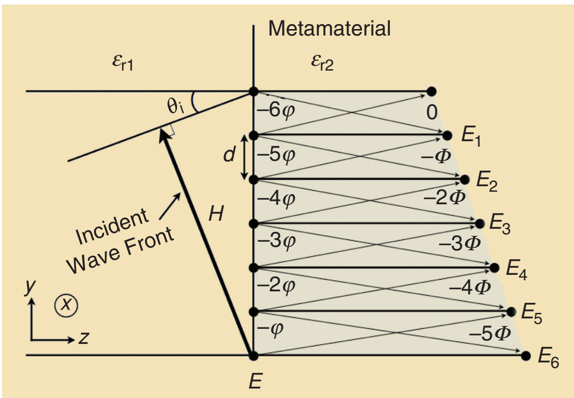

Figure 19. *Multielement metamaterial.*

In general, this means that the output from each node will form two separate wave fronts, and radiation from the output will be diffuse. Some negative and some positive refraction would be observed. However, if the angle of the output prism is such that $\Phi = 2\varphi$, then each of the two components at each output node will have the same phase. Consequently, we will have constructive addition of the two waves at each output node, and the output phases are $-6\varphi, -7\varphi, -8\varphi, \ldots$, etc.

The phase differential along the output surface is reversed with respect to the input phase front, and focused negative refraction will occur as shown in Figure 20. We see that, as a wave front travels through the metamaterial, it becomes distorted due to phase reversal between pairs of lines. The effect of the prism is to differentially delay the wave across the output so that individual phase reversed wave fronts add up constructively in the direction of negative refraction.

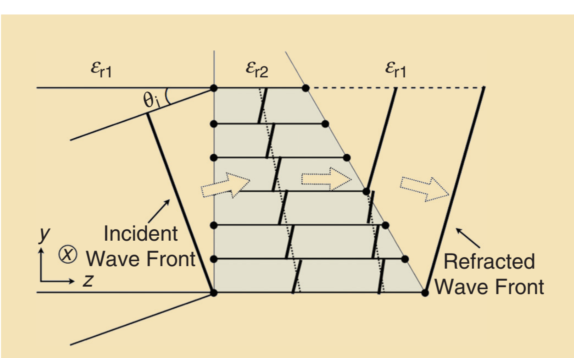

Figure 20. *Multielement metamaterial with a wave travelling through.*

Conclusion

Negative refraction cannot occur at the interface between two dielectric media, for the reason discussed in this article. However, a wide variety of lumped and distributed structures with 2-D cross sections can exhibit this phenomenon if they have finite thickness. The effect is caused by the phase difference between the even and odd modes of TE waves propagating through the metamaterial. It is also interesting to note that in both [9] and [10], the incident field is limited by passing the excitation through a slit, results in exciting only one row of nanowires, exactly analogous to the excitation of a row of parallel coupled microstrip lines, essentially an extension of the two-line pair shown in Figure 6(a), and similarly illustrated as extended to an N-line set in Figure 6(b) of this article. Figure 9(a) illustrates this excitation, while Figure 9(b) shows the result, with differing signs of refraction for TE and TM modes of excitation. This in fact is exactly what the theory presented herein explains, and the equivalent result is shown in Figure 17(b).

The theory is demonstrated by field simulations of microwave and optical structures. It is then extended to include focusing using an output prism.

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